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Electroweak Calculations In The Presence Of Nonperturbative Qcd Effects

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ELECTROWEAK CALCULATIONS IN THE PRESENCE OF NONPERTURBATIVE QCD EFFECTS

by

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ABSTRACT

A momentum space Feynman rule is derived for the quark condensate insertion into Feynman amplitudes using the quark condensate component of the nonperturbative vacuum expectation value of the two nonlocal normal ordered quark fields. The lowest-order quark condensate component of the QCD nonperturbative quark self-energy is calculated, leading to a gauge independent dynamical component for the quark mass.

Insertion of this nonperturbative order parameter into lowest-order electroweak quark self-energies is shown to satisfy on-mass-shell gauge parameter independence only if there is no contribution from the dynamical symmetry breaking parameter $<\overline{\Psi}\Psi>$. Dynamical contributions to the quark propagator, such as those from nonperturbative QCD leading to a dynamical quark mass, are shown to generate corrections to electroweak Green's functions in order to retain consistency with $SU(2)\times U(1)$ Ward identities. Such corrections lead to additional contributions to 3-and 4-point vertices which are annihilated by transverse projection operator, suggesting the utility of Landau gauge for calculations involving dynamical effects.

The induced Yukawa coupling in the chiral limit is calculated, providing an example of a physical consequence of such externally generated corrections to electroweak Green's functions.

To My Parents

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CHAPTER ONE

INTRODUCTION

1-1) Electromagnetic Interactions:

Electric and magnetic forces were known for a long time as two separate properties of matter. These two along with gravity are the only fundamental forces of nature that we can experience in our daily life. A little more than a hundred years ago Maxwell proposed the unified theory of electricity and magnetism and laid the foundation for classical electrodynamics. In this theory electric and magnetic forces are two different facets of the same force On the other hand, it was shown that electromagnetic field could sustain itself and propagate with the velocity of light.

In classical mechanics, electromagnetic interactions change the equations of motion by replacing the canonical momentum $\vec{P}$ with $\vec{P} + eQ\vec{A}$ (minimal coupling) where $\vec{A}$ is the vector potential and Q is the electric charge. The same rule (its covariant form in the relativistic case) applies to quantum mechanics. Quantum electrodynamics (QED) is the quantized version of the interaction of matter and electromagnetic field where, starting from the classical lagrangian and quantization rules (canonical or Feynman path integral formalism), different Green's functions of the theory can be obtained.

The continuous symmetries of the classical lagrangian, by Noether's theorem, lead to conserved currents and charges. For example, the conservation of electric charge requires that each term of the lagrangian must be neutral as a whole. This is equivalent to the invariance of the lagrangian $L(\partial_\mu\psi, (\partial_\mu\psi)^+, \psi, \psi^+)$ under phase shift:

$$\psi \rightarrow \psi' = \exp(ie\Lambda Q)\psi, \tag{1-1}$$

where ψ (spinor or scalar) is the charged field, Q is the charge operator and Λ is the parameter characterizing the phase transformation. The transformation (1-1) is a global U(1) transformation i.e., Λ is independent of space-time, therefore ψ and its derivatives $\partial_\mu\psi$ transform in the same way. Such is not the case for a local transformation where $\Lambda = \Lambda(x)$. Invariance under local phase shift requires more degrees of freedom which may occur by the appearance of some new fields in the theory. In fact, if we replace ∂_μ with the so called covariant derivative $D_\mu = \partial_\mu - ieQA_\mu$, it can be shown that ψ and $D_\mu\psi$ transform in the same manner under local phase shift provided the gauge field A_μ transforms as:

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu\Lambda(x). \tag{1-2}$$

Therefore, we observe that invariance under local U(1) gauge transformation necessitates minimal coupling between the gauge field and the charged field with a coupling strength eQ. We should also add the appropriate kinetic term for the gauge field A_μ, so the full lagrangian invariant under local phase transformation is:

$$L_{QED} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + L(D_\mu\psi, (D_\mu\psi)^+, \psi, \psi^+). \tag{1-3}$$

It can be shown that the equation of motion for A_μ is identical to that of Maxwell's equations, consequently, we can identify the gauge field with photon, and e with the electromagnetic coupling strength. The abelian gauge transformation which generates the electromagnetic interactions is called $U(1)_{EM}$ which has only one generator, the charge operator Q. Also we should notice that any mass term for the field A_μ violates the gauge invariance explicitly and therefore is not allowed in the QED lagrangian. This is consistent with the experimental fact that photon is massless (or, equivalently, the "inverse square law") to a very high accuracy.

In calculating the quantum corrections to the classical theory, we encounter infinities (divergent integrals) which have to be removed (renormalization)[1]. A theory is renormalizable if these infinities can be absorbed into a finite set of parameters (renormalization constants). Renormalizability is a necessary condition for theories to have predictive power. In the process of quantization, the gauge symmetry of the classical lagrangian evolves to the so called BRST (Becchi-Rouet-Stora-Taylor) symmetries[2], which in turn impose a set of relations between the Green's functions of the theory. These constraints are called Ward-Takahashi identities and have a very important role in the proof of renormalizability.

In the next section we investigate how, with the help of non-abelian gauge symmetries, a model for unified weak and electromagnetic interactions can be developed.

1-2) Weak Interactions and Electroweak Standard Model:

The story of weak interactions begins with β-decay of atomic nuclei. The slow decay rate is an indication of a new interaction much weaker than electromagnetic interaction. It was Fermi who first proposed a weak current similar to electromagnetic current leading to an effective matrix element of the form:

$$\frac{G}{\sqrt{2}}(J_{\mu}^{w})^{+}J^{w\mu}, \qquad (1-4)$$

where J_{μ}^{w} is the charged weak current and G is the Fermi constant. However, discovery of parity violation in the weak interactions by Yang and Lee in 1956[3], indicated that weak current should be of V-A (vector-axial vector) form:

$$J_{\mu}^{w} = \overline{\psi}_{i}\gamma_{\mu}(1-\gamma_{5})\psi_{l}. \qquad (1-5)$$

As a result of the charge raising (lowering) nature of charged weak current, ψ_l and ψ_i are eigenstates of charged operator whose eigenvalues differ by one.

Unlike electromagnetic coupling constant e, which is dimensionless, the Fermi coupling constant has dimension M^{-2}, a property making it impossible to calculate higher order corrections (i.e., the theory is not renormalizable). This difficulty can be avoided if we assume that a heavy massive particle mediates the weak interaction, in which case the matrix element (1-4) is the zero momentum limit of:

$$\left(\frac{g}{\sqrt{2}}\overline{\psi}_i\gamma_\mu\frac{1}{2}(1-\gamma_5)\psi_l\right)^{+}\frac{1}{M_w^2-q^2}\left(\frac{g}{\sqrt{2}}\overline{\psi}_i\gamma^\mu\frac{1}{2}(1-\gamma_5)\psi_l\right), \qquad (1-6)$$

where g is the dimensionless weak coupling constant, $M_w \approx 80GeV$ is the mass of the intermediate particle and q is the momentum carried by that particle. Also numerical coefficients have been adjusted to accommodate the appearance of the left handed projection operator $\frac{1}{2}(1-\gamma_5)$. For $q^2 \ll M_w^2$ we recover the Fermi 4-fermion point interaction with $\frac{G}{\sqrt{2}} = \frac{g^2}{8M_w^2}$. In (1-6), one sees that the weak interactions are weak at low energies not because $g \ll e$, but because of the suppression occuring when M_w is large. In fact with $g \approx e$, electromagnetic and weak interactions have comparable strength at energies $O(M_w)$ and above, a situation reminiscent of the equivalent strengths of electric and magnetic forces at high velocities.

Now we would like to generalize the idea of gauge symmetry, explained in the previous section, to non-abelian groups[4] in order to include weak interactions[5]. First of all, we observe that left-handed spinors ψ_{lL} and ψ_{iL} are connected together by the weak interaction, in which case the fundamental representation is a doublet (weak isospin):

$$\psi_L = \begin{pmatrix}\psi_l \\ \psi_i\end{pmatrix}_L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix}\psi_l \\ \psi_i\end{pmatrix}. \qquad (1-7)$$

For example, electron neutrino ν_e and electron e^- are the upper and lower components of a weak isospin doublet respectively. It is obvious that $SU(2)_L$ gauge symmetry [i.e.,

$\psi_L \to \exp(ig\Lambda^a T^a)\psi_L$ where $T^a = \frac{\sigma^a}{2}$ are the generators (σ^a a=1,2,3 are Pauli matrices)] alone can not be the symmetry of the electroweak as the generator of its U(1) subgroup, T^3, is not proportional to the charge operator. For this reason we take the direct product $SU(2)_L \times U(1)_Y$ as the symmetry group, where T° the generator of $U(1)_Y$ (hypercharge) is chosen such that its linear combination with the diagonal generator of $SU(2)_L$ would reproduce the electric charge operator i.e.:

$$\psi_L \to \exp(ig\Lambda^a T^a + ig'\Lambda^{\circ} T^{\circ})\psi_L, \tag{1-8a}$$

$$(T^3 + T^{\circ})\psi_L = Q\psi_L, \tag{1-8b}$$

where g' is the $U(1)_Y$ gauge coupling constant. The generators T^a satisfy the Lie algebra i.e., $[T^a, T^b] = i\varepsilon^{abc}T^c$ $\varepsilon^{123} = 1$, $\varepsilon^{213} = -1$ etc, and they all commute with T°. The right-handed components of the spinors $(\psi_{i,l})_R = \frac{1}{2}(1+\gamma_5)\psi_{i,l}$, which are singlets under $SU(2)_L$, transform nontrivially under $U(1)_Y$:

$$\psi_R \to \exp(ig'\Lambda^{\circ} T^{\circ})\psi_R, \tag{1-9a}$$

$$T^{\circ}\psi_R = Q\psi_R. \tag{1-9b}$$

In order to have a local gauge invariance, we have to introduce covariant derivatives for left-handed (right-handed) spinors:

$$D_{\mu L} = \partial_\mu - igW_\mu^a T^a - ig'B_\mu T^{\circ}, \tag{1-10a}$$

$$D_{\mu R} = \partial_\mu - ig'B_\mu T^{\circ}, \tag{1-10b}$$

$$W_\mu^a \rightarrow W_\mu^a + \partial_\mu \Lambda^a + g\,\varepsilon^{abc} \Lambda^b W_\mu^c, \qquad (1-10c)$$

$$B_\mu \rightarrow B_\mu + \partial_\mu \Lambda^\circ. \qquad (1-10d)$$

We observe that local gauge invariance implies dynamics involving 4 independent gauge fields W_μ^a and B_μ, and uniquely determines their couplings to the matter fields:

$$L^{fermion} = \overline{\psi}_L i\gamma^\mu D_{\mu L} \psi_L + \overline{\psi}_R i\gamma^\mu D_{\mu R} \psi_R. \qquad (1-11)$$

Moreover, the kinetic term for W_μ^a gauge fields is slightly different from abelian case because of the non-abelian nature of $SU(2)_L$, particularly the nonvanishing commutators of its generators:

$$L_{kinetic}^{gauge} = -\frac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\,\varepsilon^{abc} W_\mu^b W_\nu^c)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2. \qquad (1-12)$$

The non-abelian term leads to self-coupling of W fields among themselves, a property not shared by abelian gauge fields (i.e. photon).

As indicated earlier, the mediators of the weak interaction ought to be massive. We have already argued, however, that any mass term for gauge fields violates the gauge symmetry (as in QED). For chiral theories (unlike QED), even fermions are not allowed to be massive; a mass term, $m\overline{\psi}\psi = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$, couples left and right-handed spinors and therefore is not invariant under $SU(2)_L$ or $U(1)_Y$. In quantum field theory there is a method for generating mass for the gauge fields and fermions without breaking the underlying symmetry explicitly. This method is based on the idea of spontaneous symmetry breakdown[6]. Whenever we are dealing with infinite number of degrees of freedom, like in quantum field theory, it is possible to have degenerate vacuum states (the minimum energy states) which uphold the symmetry of the lagrangian. But the real world can have only one vacuum, which is not necessarily invariant under the full lagrangian symmetry. When the true vacuum fails to uphold the full symmetry of the lagrangian, massless scalar fields occur, Nambu-Goldstone (NG) bosons (one for every broken generator), as the remnants of the underlying symmetry. In other words, by picking out one vacuum state as the real world vacuum, the existence of other degenerate vacuum states manifests itself as massless excitations to this ground state.

Spontaneous symmetry breakdown can be used in conjunction with local gauge symmetries to generate mass for the gauge fields. This is the so called Higgs mechanism[7] in which the NG bosons become unphysical by being absorbed as the longitudinal part of the gauge fields, thereby permitting these fields to be massive. The advantage of this method of mass generation is that the symmetry of the lagrangian, eventhough not explicit, remains intact, and the theory retains its renormalizability[8]. To show how this idea works for electroweak model, we add a scalar part to the lagrangian which is invariant under $SU(2)_L \times U(1)_Y$:

$$L^{scalar} = (D_{\mu L}\Phi)^{+} D_L^{\mu}\Phi - V(\Phi), \qquad (1-13a)$$

$$V(\Phi) = \mu^2\Phi^{+}\Phi + \lambda(\Phi^{+}\Phi)^2, \qquad (1-13b)$$

$$\Phi \to \exp(ig\Lambda^a T^a + ig'\Lambda^o T^o)\Phi \qquad (1-13c)$$

where $\Phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$ is called the Higgs field and λ has to be positive for energy to be bounded from below. In order to find the vacuum configuration we may rewrite the potential by completing the square:

$$V(\Phi) = \lambda\left(\Phi^{+}\Phi + \frac{\mu^2}{2\lambda}\right)^2 - \frac{\mu^4}{4\lambda}. \qquad (1-14)$$

For negative μ^2 there are nontrivial vacua satisfying $\langle\Phi\rangle^{+}\langle\Phi\rangle = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$. To pick one of these states as our physical vacuum we should keep in mind that it must be invariant under $U(1)_{EM}$ [i.e., the electric charge operator must still annihilate the physical vacuum to uphold the physically observed conservation of electric charge]. In our representation for charge operator (1-8b) i.e., $Q = \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}$, we can write the physical vacuum as one supporting a nonzero scalar-field vacuum expectation value:

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}. \qquad (1-15)$$

In other words, our choice of physical vacuum only upholds the $U(1)_{EM}$ subgroup of the Lagrangian's overall $SU(2)_L \times U(1)_Y$ symmetry. Now we express the field Φ in terms of its deviations from the vacuum expectation value (1-15):

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} i\chi_1 + \chi_2 \\ v + \phi - i\chi_3 \end{pmatrix}, \tag{1-16}$$

where three components of Φ i.e., χ_1, χ_2 and χ_3 are the unphysical massless NG bosons that appear due to the spontaneous breakdown of the three of four generators, and where ϕ is the remaining massive physical Higgs field with mass $m_\phi = \sqrt{2}\mu$. Because of the local gauge symmetry of the lagrangian, one is able to obtain a gauge transformation involving the broken generators that transforms these unphysical fields away (unitary gauge):

$$\Phi \rightarrow U\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \phi \end{pmatrix}. \tag{1-17}$$

Therefore three of the four gauge fields ($W^{1,2,3}, B$) appearing in the kinetic term of (1-13a) via (1-10) acquire bilinear mass terms proportional to scalar field vacuum expectation value (VEV) v. To relate these gauge fields to the charged $W^{\pm}$, neutral (Z) and photon (A) fields of electroweak interactions, we define :

$$W_\mu^{\mp} = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2), \tag{1-18a}$$

$$Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu, \tag{1-18b}$$

$$A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu, \tag{1-18c}$$

where θ_w, the Weinberg angle, along with electromagnetic coupling e, are related by the requirement that A_μ couple to electromagnetic current:

$$\sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}, \qquad (1-19a)$$

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}. \qquad (1-19b)$$

No mass term arises for A_μ. Z_μ and $W_\mu^\pm$ masses, however, do arise, and can be expressed in terms of g, g' and v:

$$M_w = \frac{1}{2} g v, \qquad (1-20a)$$

$$M_z = \frac{1}{2}\sqrt{g^2+g'^2}\, v = \frac{M_w}{\cos\theta_w}. \qquad (1-20b)$$

In order to generate mass for fermions, we introduce gauge invariant coupling between scalar Higgs field and fermions (Yukawa coupling):

$$L_Y = -g_Y^i \overline{\Psi}_L \Phi \Psi_R + h.c., \qquad (1-21)$$

where g_Y^i is the Yukawa coupling constant, a free parameter of the theory. When Φ develops a nonzero VEV, a mass term for the lower component of the fermion doublet is generated:

$$L_{mass} = -g_Y^i \frac{v}{\sqrt{2}} \overline{\Psi}_i \Psi_i. \qquad (1-22)$$

The upper component remains massless, consistent with the SM assumption of having massless neutrinos in the lepton sector. On the other hand, if we try to generalize the Yukawa coupling to the quark sector where both components of the fermion doublet are massive, we must introduce a new Yukawa-interaction term:

$$L_Y^H = -g_{Yab}^i \overline{\Psi}_L^a \Phi \Psi_R^b - g_{Yab}^{\prime} \overline{\Psi}_L^a (i\sigma_2 \Phi^*) \Psi_R^b + h.c.. \qquad (1-23)$$

In the quark model, where the lightest quarks [i.e., up and down quarks] form an electroweak doublet, we usually refer to lepton and quark doublets together as a family. In (1-23) a and b are family indices. There are two more families consisting of heavier quarks and leptons:

$$\begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} u \\ d \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{pmatrix}. \qquad (1-24)$$

Differing masses provide the only difference between these families from electroweak interaction point of view. Strangeness changing weak decays are an indication of mixing in the hadronic sector of different families. As we see in (1-23), quark weak eigenstates appearing in the lagrangian are mixtures of quark mass eigenstates. The so called Kobayashi-Maskawa (K-M)[9] mixing matrix is the unitary matrix arises from diagonalizing the mass matrices generated through (1-23) when $<\Phi>\neq 0$.

In the process of quantization, two more terms are added to the classical lagrangian. One of these is the gauge fixing term which is needed to remove redundant gauge degrees of freedom. The coefficient of this term, the gauge parameter (GP), is arbitrary; any meaningful physical observable in the theory must be gauge parameter independent (GPI). As we mentioned earlier, unphysical NG bosons may all be removed by choosing unitary gauge. Unfortunately perturbative calculations in this gauge are not demonstrably renormalizable. Instead, it is customary to work in 'tHooft-Feynman gauge, in which gauge fields are accompanied by their unphysical scalar partners. This gauge is called R-gauge or renormalizable gauge[10]. The seccnd term required for quantization is the Fadeev-Popov ghost term[11], which is required to preserve unitary and gauge invariance in calculations involving loops. Ghosts are scalar fields satisfying Fermi statistics, therefore they never occur as initial or final states in physical processes

Now we can write the full electroweak lagrangian:

$$L_{electroweak} = L^{gauge}_{kinetic} + L^{fermion} + L^{scalar} + L^{Y} + L^{gauge-fixing} + L^{ghost}. \qquad (1-25)$$

Some of the electroweak Feynman rules derived from this lagrangian are listed in Appendix 1. Appendix 2 is a list of BRST transformations that leave this lagrangian invariant.

1-3) Quantum Chromodynamics (QCD)

QCD is the theory of the strong interactions. It is based on $SU(3)_c$ nonabelian color gauge symmetry. Particles with color charge can not be detected in isolation (confinement) and observed hadrons are postulated to be $SU(3)_c$ singlet, a result of the increasing strength of strong interactions at long distances. However, there is substantial evidence for believing in color charge[12]:

1) Baryons consist of three quarks, therefore there must be an attractive force for the state qqq. It can be shown that $SU(3)_c$ leads to an attractive potential between any two quarks in a baryon (color singlet state)[13].

2) The spin $\frac{3}{2}$ baryon Δ^{++}, which is a member of a completely symmetric decouplet, consists of three up quarks in their ground states. A color quantum number[14] is needed to anti-symmetrize its (otherwise symmetric) wavefunction.

3) The decay process $\pi^0 \rightarrow 2\gamma$ occurs mainly through a quark loop coupled to a pion on one side and to two photons on the other side (fig.1). The total amplitude is independent of the intermediate quark masses[15] and must be the sum over all quarks that couple to pion[16]. Thus this amplitude is proportional to the number of colors N_c. The experimental amplitude is consistent with $N_c = 3$[17].

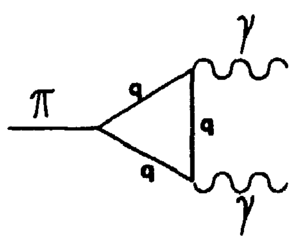


Figure 1: Decay of the neutral pion into two photons via a quark loop.

4) The cross section for electron-positron annihilation producing hadrons is also proportional to N_c. Comparison with experiment requires that $N_c = 3$[18].

The gauge theory of strong interactions based on $SU(3)_c$ requires that QCD lagrangian be invariant under the transformation:

$$\Psi_i \to \left(\exp\left(ig_s\Lambda^a\frac{\lambda^a}{2}\right)\right)_{ij}\Psi_j, \qquad (1-26)$$

where λ^a a=1..8 are $SU(3)$ generating matrices and i,j=1,2,3 are color indices. Again, in order to have local gauge symmetry, we have to introduce gauge fields to construct covariant derivatives. The classical lagrangian can be written as:

$$L_{QCD} = \overline{\Psi}_i i\gamma^\mu (D_\mu)_{ij}\Psi_j - m^c\overline{\Psi}\Psi - \frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c)^2, \qquad (1-27a)$$

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s A_\mu^a\left(\frac{\lambda^a}{2}\right)_{ij}, \qquad (1-27b)$$

where A_μ^a a=1..8 are the gluon fields and f^{abc} is the $SU(3)$ group structure constant. m^c is the lagrangian quark mass also known as current quark mass.

It is an experimental fact that quark substructure of hadrons (like nucleons) manifests itself at short distances; i.e., at large momenta, QCD binding forces become weak enough that quarks behave like free particles[19]. On the other hand, as we mentioned earlier, at long distances (or low momentum regime) color forces are sufficiently strong that hadrons are confined to a finite size and therefore they appear structureless at distances longer than one Fermi.

The renormalized strong coupling constant g_s^r, calculated to lowest perturbative order (one loop) has inverse logarithmic dependence on momentum which indicates its small value at large momentum (compared to $\Lambda_{\overline{MS}}$, a QCD scale parameter arising from the renormalization process). As the perturbative expansion converges only when $\alpha_s = \frac{g_s^2}{4\pi} < O(1)$, the usual order-by-order perturbative calculation technique breaks down at low momenta (nonperturbative region). With the help of huge computers, a technique known as lattice gauge theory, employing the QCD lagrangian as input has been developed in which all hadronic spectra can be calculated to a precision limited only by the size of the computing machine[20]. This indicates the validity of (1-27) even in the nonperturbative region.

The current mass m^c is a free parameter in L_{QCD} and its origin could be from the spontaneous symmetry breaking of the electroweak lagrangian [i.e., m^c is generated by the nonzero VEV of the Higgs field $< \Phi >$]. This is the mass parameter attributed to quarks in deep inelastic scatterings (like eN or νN inelastic scattering). Its value for up and down quarks, which is also consistent with the lattice gauge calculations[21], is in the range $m^c_{u,d} \approx 5 - 10$Mev. On the other hand, there is another quark mass parameter which appears in nonrelativistic quark model and hadronic spectroscopy. For example with quarks being fundamental Dirac particles, the nucleon magnetic moment is obtained by assuming

$m_{u,d} \approx \frac{M_{nucleon}}{3} \approx 300\text{Mev}$[22]. This mass parameter is known as the constituent quark mass. It is believed that the gap between m^c and m is the result of dynamical chiral symmetry breaking of due to QCD.

Since the mass term in (1-27a) is the only coupling between right-handed and left-handed spinors, i.e. $\overline{\Psi}\Psi = \overline{\Psi}_L\Psi_R + \overline{\Psi}_R\Psi_L$, Ψ_L and Ψ_R can be transformed independently in the $m^c = 0$ limit, or equivalently, the lagrangian is invariant under both $U(1)_V$ and $U(1)_A$ (V and A refer to vector and axial vector transformations). This is known as chiral symmetry and $m^c = 0$ is called the chiral limit. As the u and d current masses are much smaller than the scale of color interactions (~200Mev), chiral $SU(2)_R \times SU(2)_L$ symmetry (in u and d flavour space) is upheld approximately by the QCD lagrangian. Now if we further assume that the QCD vacuum is chiral noninvariant and breaks this chiral symmetry spontaneously, generating a dynamical mass term (while keeping isospin intact), then according to Goldstone theorem[23] there must be three massless NG bosons due to the breakdown of three generators:

$$SU(2)_R \times SU(2)_L \rightarrow SU(2)_V. \qquad (1-28)$$

The near masslessness of the three pions relative to all other hadronic states together with the almost exact isospin symmetry observed in nature, indicates that the above assumption is reasonable.

It is possible to parametrize the nonperturbative QCD vacuum in terms of condensates, vacuum expectation values of products of fields. These parameters are classified according to their mass dimension D. For example, D=3 quark condensate $<\overline{\Psi}\Psi>$, D=4 gluon condensate $<F^a_{\mu\nu}F^a_{\mu\nu}>$ and D=5 mixed condensate $<\overline{\Psi}\sigma.G\Psi>$ are the first three condensates of lowest mass dimension. A simple dimensional argument indicates that condensates become less important as their mass dimension increases. Therefore, at high enough momenta the dominant chiral symmetry breaking conter the QCD vacuum can be approximated in terms of its quark condensate component. Indeed nonperturbative QCD vacuum permits the existence of a nonzero VEV for operators like $\overline{\Psi}\Psi$ which are not chirally invariant. In fact using a PCAC (Partially Conserved Axial Current) argument[24], it can be shown that $-(m^c_u + m^c_d)(<\overline{u}u> + <\overline{d}d>) = f^2_\pi m^2_\pi$ which implies a nontrivial condensate.

One way of calculating the dynamical component of quark mass generated by the QCD vacuum is to allow the nonperturbative VEV's of the normal ordered product of fields, which are identically zero in the perturbative field theory, to enter Schwinger-Dyson series of the quark two point function. Hence by extracting different condensate components of these VEV's, known as operator product expansion (OPE), we are able to find contributions from these various condensates in the quark self-energy. The presence of these nonperturbative self-energies may shift the quark propagator pole from its lagrangian value m^c, thereby generating a dynamical component for quark mass.

The interpretation of the propagator pole as the physical mass (constituent quark mass) is open to discussion because quarks do not exist as asymptotically free particles due to confinement. But if we assume that the dynamical chiral symmetry breaking and confinement are related to different QCD order parameters, then it is reasonable to study the dynamical mass generation in the absence of an explicit confinement mechanism.

As the physical quark mass must be independent of arbitrary gauge parameters in order to be acceptable as a physical observable, quark self-energies are required to be GPI on mass shell. This requirement is satisfied for purely perturbative self-energies due to the gauge invariance of the lagrangian. However, as the full standard model has the gauge symmetry of $SU(3)_c \times SU(2)_L \times U(1)_Y$, the chiral-symmetry-violating QCD-vacuum effects have to be extended into the electroweak theory. This requires a careful treatment. For example, the QCD-generated dimension-3 quark condensate is not invariant under $SU(2)_L$ ($\vee$ $U(1)_Y$), and consequently, its insertion into electroweak processes breaks the gauge symmetry explicitly.

As we mentioned earlier, Ward identities (known as Slavnov-Taylor identities in nonabelian theories)[25] are the consequences of the local gauge symmetry of the lagrangian and must be upheld in order to have GPI S matrix. Therefore a consistent insertion of QCD generated nonperturbative self-energies into electroweak interactions may be possible if

$SU(2)_L \times U(1)_Y$ Ward identities are imposed on the QCD-enhanced Green's functions. Hence, in this way the presence of the QCD nonperturbative order parameters may enter electroweak Feynman rules.

The main focus of this thesis is on how to treat nonperturbative QCD effects in electroweak calculations. In our approach, we consider $SU(2)_L \times U(1)_Y$ in isolation from $SU(3)_c$ and investigate the role of externally (i.e., $SU(3)_c$) generated chirally noninvariant self-energies in electroweak processes.

Chapter Two of this thesis is devoted to the role of the quark condensate component of the quark's self-energy. First, the quark condensate component of the nonlocal normal ordered product of two quark fields, derived by Elias, Steele and Scadron[26], will be used to obtain a momentum space Feynman rule for the quark condensate insertions in a Feynman diagram. This enables calculation of the $<\bar{\Psi}\Psi>$ component of the quark self-energies mediated by a single gluon exchange.

In Chapter Three, gauge parameter dependence of $<\bar{\Psi}\Psi>$ contributions to electroweak (W, A and Z mediated) self-energies is considered. This gauge parameter dependence is shown to vanish on the quark mass shell, if that mass shell is identified throughout with the propagator pole.

In Chapter Four, a more general approach is adopted by assuming that a momentum-dependent externally generated chiral-symmetry-violating self-energy enters electroweak calculations via the quark propagator. Corrections to the electroweak Green's functions are obtained through use of $SU(2)_L \times U(1)_Y$ Ward identities. The physical consequences of these corrections are then investigated.

CHAPTER TWO

$<\overline{\Psi}\Psi>$ CONTRIBUTIONS TO QUARK SELF-ENERGIES

2-1) The quark condensate Feynman rule:

Nonperturbative QCD vacuum which breaks chiral symmetry leads to a nonzero VEV for the nonlocal normal ordered product of two quark fields, thereby providing additional Wick-expansion contributions to the S matrix. The quark condensate component of this nonperturbative VEV has been calculated through the use of the operator product expansion method[26]:

$$<0\,|:\overline{\Psi}(x)\Psi(0):|\,0>^{NP}=\frac{<\overline{\Psi}\Psi>}{6m}\left(\frac{J_1(m\sqrt{x^2})}{\sqrt{x^2}}+\frac{i\gamma.xJ_2(m\sqrt{x^2})}{x^2}\right), \qquad (2-1)$$

where J_1 and J_2 are Bessel functions.

Now as in the perturbative case where the quark propagator in momentum space is given by

$$S(p)=(m^c-\gamma.p)^{-1}=i\int d^4(x-y)\exp(-ip.(x-y))<0\,|\,T\Psi(x)\overline{\Psi}(y)\,|\,0>^{pert.}, \quad (2-2)$$

we may find a momentum space Feynman rule for the nonperturbative quark condensate:

$$S^{NP}(p)=-i\int d^4x\exp(-ip.x)<0\,|:\overline{\Psi}(x)\Psi(0):|\,0>^{NP}. \qquad (2-3)$$

Substituting for nonperturbative VEV from (2-1) yields:

$$S^{NP}(p)=\frac{-i<\overline{\Psi}\Psi>}{6m}\int d^4x\exp(-ip.x)\left(\frac{J_1(m\sqrt{x^2})}{\sqrt{x^2}}+\frac{i\gamma.xJ_2(m\sqrt{x^2})}{x^2}\right)$$

$$=-\frac{i<\overline{\Psi}\Psi>}{6m}\int dq^2\delta(q^2-1)(I_1+I_2), \qquad (2-4a)$$

$$I_1=\int d^4x\exp(-ip.x)\frac{J_1(m\sqrt{q^2x^2})}{\sqrt{q^2x^2}}, \qquad (2-4b)$$

$$I_2=i\int d^4x\exp(-ip.x)\frac{\gamma.xJ_2(m\sqrt{q^2x^2})}{x^2}. \qquad (2-4c)$$

I_1 is calculated by rotation to Euclidean space:

$$I_1=i\int_E d^4x\exp(ip.x)\frac{J_1(qmx)}{qx}$$

$$=\frac{4\pi i}{q}\int_0^\infty dxx^3\frac{J_1(qmx)}{x}\int_0^\pi d\theta\sin^2\theta\exp(ipx\cos\theta). \quad (2-5)$$

where $p=\sqrt{p_E^2}, q=\sqrt{q_E^2}$ and $x=\sqrt{x_E^2}$. The angle integration can be performed using the formula[27]:

$$\int_0^\pi d\theta\exp(\pm ipx\cos\theta)\sin^2\theta=\frac{\pi J_1(px)}{px}. \qquad (2-6)$$

Substituting (2-6) in (2-5) and using orthogonality properties of the Bessel functions we obtain:

$$I_1 = \frac{4\pi^2 i}{qp}\int_0^\infty dx\, x J_1(qmx) J_1(px)$$

$$= \frac{4\pi^2 i}{qp^2}\delta(p - qm). \qquad (2-7)$$

I_2 is calculated using the relation $\gamma.x = -i\gamma.\frac{\partial}{\partial p}$ in the appropriate Euclidean-space expression:

$$I_2 = -\int_E d^4x \frac{\gamma.x J_2(qmx)}{x^2}\exp(ip.x)$$

$$= 4\pi i\gamma.\frac{\partial}{\partial p}\int_0^\infty dx\, x^3 \frac{J_2(qmx)}{x^2}\int_0^\pi d\theta \sin^2\theta \exp(ipx\cos\theta).$$

$$(2-8)$$

Again using (2-6), the angle integration is performed:

$$I_2 = 4\pi^2 i\gamma.\frac{\partial}{\partial p}\int_0^\infty dx\, J_2(qmx)\frac{J_1(px)}{p}. \qquad (2-9)$$

Utilizing the Bessel function's property $\gamma.\frac{\partial}{\partial p}\left(\frac{J_1(px)}{p}\right) = \frac{-\gamma.p}{p^2} x J_2(px)$, one finds that

$$I_2 = -\frac{4\pi^2 i\gamma.p}{p^2}\int_0^\infty dx\, x J_2(qmx) J_2(px)$$

$$= -\frac{4\pi^2 i}{qmp^2}\gamma.p\,\delta(p - qm). \qquad (2-10)$$

Substitution of (2-7) and (2-10) into (2-4) yields

$$I_1 + I_2 = \frac{4\pi^2 i}{qmp^2}(-\gamma.p + m)\delta(p - qm)$$

$$= \frac{8\pi^2 i}{p^2}(-\gamma.p + m)\delta(p^2 - q^2m^2) \qquad (2-11)$$

Now transforming back to the Minkowski space we obtain the momentum space representation of the quark condensate component of the nonperturbative VEV of the normal ordered product of the two quark fields[28]:

$$S^{NP}(p) = -\frac{4\pi^2 < \overline{\Psi}\Psi >}{3mp^2}(\gamma.p + m)\delta(p^2 - m^2)$$

$$= -\frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3}(\gamma.p + m)\delta(p^2 - m^2). \qquad (2-12)$$

The appearance of delta function in the Feynman rule simplifies loop calculations involving the quark condensate.

I would like to add a cautionary note at this point. The prescription introduced in this section (see equation (2-4)) to enable us to rotate into and back from Euclidean space, is not mathematically rigorous. The reason for introducing the auxiliary momentum q^2 is to make the integrals well-defined in Euclidean space. On the other hand, as will be seen in the next section, in order to pick up a nonzero contribution from the insertion of S^{NP} into a loop calculation, we use this prescription again as follows: First one rotates back to Minkowski space and then one integrates over q^2. Changing the order of this procedure would lead to a *different* result. I have not been able to resolve this ambiguity so far but I am actively investigating this point.

2-2) $<\overline{\Psi}\Psi>$ component of the quark self-energy due to a gluon exchange:

As an example of QCD generated nonperturbative self-energy, we calculate the lowest order $O(g_s^2)$ quark condensate contribution to the quark self-energy (fig.2).

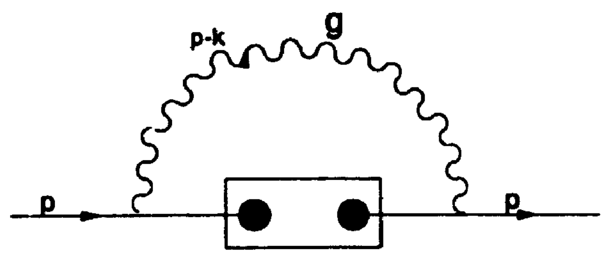


Figure 2: The lowest order quark condensate contribution to the QCD quark self-energy.

Using the Feynman rule for the quark condensate (2-12), one finds this self-energy to be

$$\sigma(p)=\int\frac{d^4k}{i(2\pi)^4}\left(g_s\gamma^\mu\frac{\lambda^a}{2}\right)\left[-\frac{4\pi^2<\overline{\Psi}\Psi>}{3m^3}(\hat{k}+m)\delta(k^2-m^2)\right]$$

$$\left(g_s\gamma_\nu\frac{\lambda^a}{2}\right)\left(\frac{g_{\mu\nu}}{(p-k)^2}-(1-\xi_g)\frac{(p-k)_\mu(p-k)_\nu}{(p-k)^4}\right)$$

$$=g_s^2\left(\frac{\lambda^a}{2}\frac{\lambda^a}{2}\right)\frac{4\pi^2<\overline{\Psi}\Psi>}{i(3m^3)(2\pi)^4}\int d^4k(\frac{2\hat{k}-4m}{(p-k)^2}$$

$$+(1-\xi_g)\frac{(\hat{p}-\hat{k})(\hat{k}+m)(\hat{p}-\hat{k})}{(p-k)^4})\delta(k^2-m^2), \qquad (2-13)$$

where ξ_g is the gauge parameter associated with gluon and $\hat{k}=\gamma.k$. Using the relation $\frac{\lambda^a}{2}\frac{\lambda^a}{2}=\frac{4}{3}$ and replacing $\hat{k}\rightarrow\frac{\hat{p}}{p^2}p.k$ yields

$$\sigma(p)=\int dq^2 f(p,q^2)\delta(q^2-1) \qquad (2-14a)$$

$$f(p,q^2)=\frac{g_s^2<\overline{\Psi}\Psi>}{9i\pi^2m^3}\int d^4k[\frac{\frac{2\hat{p}p.k}{p^2}-(3+\xi_g)m}{(p-k)^2}$$

$$+(1-\xi_g)\frac{\frac{(p^2+k^2)\hat{p}p.k}{p^2}-2k^2\hat{p}}{(p-k)^4}]\delta(k^2-q^2m^2), \qquad (2-14b)$$

which in Euclidean space becomes

$$f(p,q^2)_E=\frac{4g_s^2<\overline{\Psi}\Psi>}{9\pi m^3}\int_0^\infty dkk^3\int_0^\pi d\theta\sin^2\theta[\frac{\frac{2\hat{p}pk}{p^2}\cos\theta+(3+\xi_g)m}{(p^2+k^2-2pk\cos\theta)}$$

$$+(1-\xi_g)\frac{\frac{(p^2+k^2)\hat{p}pk}{p^2}\cos\theta-2k^2\hat{p}}{(p^2+k^2-2pk\cos\theta)^2}]\delta(k^2-q^2m^2). \qquad (2-15)$$

Now we use the following integration formulae:

$$\int_0^\pi d\theta \frac{\sin^2\theta}{(p^2+k^2-2pk\cos\theta)} = \pi\frac{p^2+k^2}{4p^2k^2}\left(1-\left|\frac{p^2-k^2}{p^2+k^2}\right|\right), \qquad (2-16a)$$

$$\int_0^\pi d\theta \frac{\sin^2\theta\cos\theta}{(p^2+k^2-2pk\cos\theta)} = \frac{\pi}{4pk}\left(-1+\frac{(p^2+k^2)^2}{2p^2k^2}\left(1-\left|\frac{p^2-k^2}{p^2+k^2}\right|\right)\right), \qquad (2-16b)$$

$$\int_0^\pi d\theta \frac{\sin^2\theta}{(p^2+k^2-2pk\cos\theta)^2} = \frac{\pi}{4p^2k^2}\left(-1+\left|\frac{p^2-k^2}{p^2+k^2}\right|+\frac{4p^2k^2}{(p^2+k^2)^2}\left|\frac{p^2+k^2}{p^2-k^2}\right|\right),$$

$$(2-16c)$$

$$\int_0^\pi d\theta \frac{\sin^2\theta\cos\theta}{(p^2+k^2-2pk\cos\theta)^2} = \pi\frac{p^2+k^2}{4p^3k^3}\left(-1+\left|\frac{p^2-k^2}{p^2+k^2}\right|+\frac{2p^2k^2}{(p^2+k^2)^2}\left|\frac{p^2+k^2}{p^2-k^2}\right|\right),$$

$$(2-16d)$$

to perform the angle integration in (2-15):

$$\begin{aligned} f(p,q^2)_E = \frac{g_s^2 <\overline{\Psi}\Psi>}{18m^3}\int_0^\infty dk k^2 [(3+\xi_g)m\,\frac{p^2+k^2}{p^2k^2}\left(1-\left|\frac{p^2-k^2}{p^2+k^2}\right|\right) \\ +\frac{2\hat{p}}{p^2}\left(-1+\frac{(p^2+k^2)^2}{2p^2k^2}\left(1-\left|\frac{p^2-k^2}{p^2+k^2}\right|\right)\right) \\ +(1-\xi_g)\{-2\frac{\hat{p}}{p^2}\left(-1+\left|\frac{p^2-k^2}{p^2+k^2}\right|+\frac{4p^2k^2}{(p^2+k^2)^2}\left|\frac{p^2+k^2}{p^2-k^2}\right|\right) \\ +\hat{p}\frac{(p^2+k^2)^2}{p^4k^2}\left(-1+\left|\frac{p^2-k^2}{p^2+k^2}\right|+\frac{2p^2k^2}{(p^2+k^2)^2}\left|\frac{p^2+k^2}{p^2-k^2}\right|\right)\}]\delta(k^2-q^2m^2). \end{aligned}$$

$$(2-17)$$

Therefore the self-energy in Minkowski space can be written as

$$\sigma(p) = -\frac{g_s^2 <\overline{\Psi}\Psi>}{18m}\{-(3+\xi_g)m\frac{p^2+m^2}{p^2m^2}\left(1-\left|\frac{p^2-m^2}{p^2+m^2}\right|\right)$$

$$+\frac{2\hat{p}}{p^2}\left(-1+\frac{(p^2+m^2)^2}{2p^2m^2}\left(1-\left|\frac{p^2-m^2}{p^2+m^2}\right|\right)\right)$$

$$+(1-\xi_g)\{-2\frac{\hat{p}}{p^2}\left(-1+\left|\frac{p^2-m^2}{p^2+m^2}\right|+\frac{4p^2m^2}{(p^2+m^2)^2}\left|\frac{p^2+m^2}{p^2-m^2}\right|\right)$$

$$+\hat{p}\frac{(p^2+m^2)^2}{p^4m^2}\left(-1+\left|\frac{p^2-m^2}{p^2+m^2}\right|+\frac{2p^2m^2}{(p^2+m^2)^2}\left|\frac{p^2+m^2}{p^2-m^2}\right|\right)\}\},$$

which simplifies to

$$\sigma(p) = \frac{g_s^2 <\overline{\Psi}\Psi>}{9p^2}\left[(3+\xi_g)-\xi_g\frac{m\hat{p}}{p^2}\right]\Theta(|p^2|-m^2)$$

$$+\frac{g_s^2 <\overline{\Psi}\Psi>}{9m^2}\left[(3+\xi_g)-\xi_g\frac{\hat{p}}{m}\right]\Theta(m^2-|p^2|), \qquad (2-18)$$

where Θ is the Heaviside step function. We delineate three important points about (2-18) worth noting:

i) In the limit $\hat{p} \rightarrow m$ the self-energy is independent of the gluon gauge parameter, which justifies the identification of m, OPE mass parameter, with the quark physical mass.

ii) This method of calculating the nonperturbative QCD effects is based on factorization, i.e. we assume that condensates are the result of nonperturbative QCD vacuum but the coefficients of their contributions are calculated perturbatively. Therefore the results are not to be extended below some momentum threshold where perturbative QCD is not reliable. However, the self-energy (2-18) which is valid for deep Euclidean region shows a reasonable behavior even for small momenta (fig. 3).

iii) The expression (2-18) agrees in the small momentum region with the expression obtained by Reinders and Stam[29] using plane wave methods. This agreement appears to confirm the equivalence of calculations utilizing the propagator (2-12) with the more usual approach of averaging over plane wave states.

iv) For negative Minkowski p^2 the self-energy remains flat at the value $\frac{g_s^2 <\bar{\Psi}\Psi>}{3m^2}$ for $p^2 \geq -m^2$. At this point there is a discontinuity and $\sigma = \frac{g_s^2 <\bar{\Psi}\Psi>}{3p^2}$ for $p^2 < -m^2$. There are two aspects of our results that do not agree with previous studies. First, there is no physical reason for the self-energy to be discontinuous at $p^2 = -m^2$. Indeed, the Schwinger-Dyson equation studies lead to no such discontinuity for σ. Second, both OPE calculations and Schwinger-Dyson equation investigations show that $\sigma \sim A/p^2$ (up to logarithmic corrections) in the $p^2 \to -\infty$ limit where A is a positive quantity. As mentioned at the end of section 2-1, had the integration over k^2 been performed in Minkowski space, i.e. in (2-17), taking $q^2 = 1$ to pick up a nonzero contribution, we would have obtained a result different from

(2-18) and fig. 3 in that p^2 should be replaced by $-p^2$. This form of the result would agree with previous studies mentioned above both at $p^2 = -m^2$ and for $p^2 \to -\infty$. I expect this ambiguity to be resolved shortly.

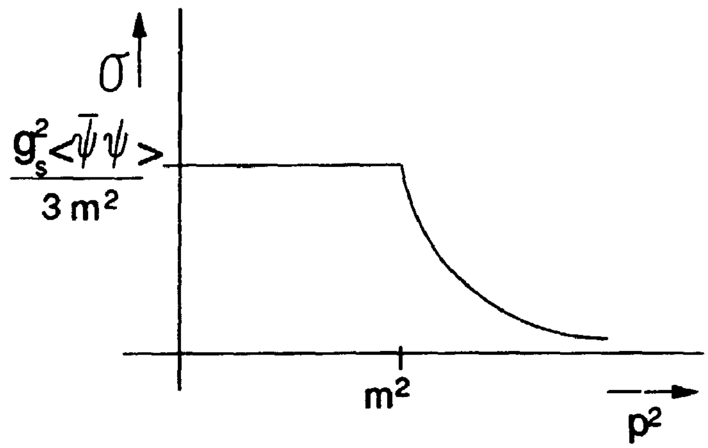


Figure 3: $\langle \bar{\Psi}\Psi \rangle$ contribution to one loop quark self-energy in Landau ($\xi_g = 0$) gauge.

As we mentioned earlier, the nonperturbative self-energy (2-18) shifts the quark propagator pole from its lagrangian value:

$$S^{-1}(p) = m^c - \hat{p} - \sigma(p)$$

$$= m^c - \hat{p} - \frac{g_s^2 < \overline{\Psi}\Psi >}{2p^2}\left[(3+\xi_g) - \frac{\xi_g m \hat{p}}{p^2}\right]$$

$$= \left(1 - \frac{\xi_g m g_s^2 < \overline{\Psi}\Psi >}{9p^4}\right)\left[\frac{m^c - \frac{g_s^2 < \overline{\Psi}\Psi >}{9p^2}(3+\xi_g)}{1 - \frac{\xi_g m g_s^2 < \overline{\Psi}\Psi >}{9p^4}} - \hat{p}\right]$$

(2 − 19)

for $p^2 > m^2$. This leads to a running mass

$$M(p^2) = \frac{m^c - \frac{g_s^2 < \overline{\Psi}\Psi >}{9p^2}(3+\xi_g)}{1 - \frac{\xi_g m g_s^2 < \overline{\Psi}\Psi >}{9p^4}}. \qquad (2-20)$$

The requirement that m be the physical quark mass (propagator pole), i.e. $M(m^2) = m$, yields a gauge parameter independent mass relation[26]:

$$m = m^c - \frac{g_s^2 < \overline{\Psi}\Psi >}{3m^2}. \qquad (2-21)$$

The second term in (2-21) provides a dynamical mass component for the quark mass in the chiral limit $m^c \to 0$[30]:

$$m_{dyn} = \left[\frac{-g_s^2 < \overline{\Psi}\Psi >}{3}\right]^{1/3}. \qquad (2-22)$$

The dynamical mass estimated from (2-22), $m_{dyn} \approx 320\text{Mev}$ [using $\alpha_s = \frac{g_s^2}{4\pi} \approx 0.5$ and $<\overline{\Psi}\Psi> \approx (-250\text{Mev})^3$ at 1 Gev momentum transfer] is in good agreement with phenomenological expectations, as discussed by Elias, Scadron and Tong[31].

From (2-21) we observe that the physical quark mass m has a component due to the chiral symmetry breaking parameter $<\overline{\Psi}\Psi>$, in addition to the current mass m^c, generated by the electroweak spontaneous symmetry breaking parameter $<\Phi>$. The GPI of (2-18) on the $\hat{p} = m$ mass shell can be interpreted as a manifestation of the GPI of any S-matrix amplitude between physical states, including even the asymptotic single fermion states characterizing a quark two point function. Of course such an interpretation of the on shell gauge independence of (2-18) is debatable, as quarks in QCD ultimately cannot correspond to asymptotic free particle states because of confinement. To test convincingly the on shell gauge independence of $<\overline{\Psi}\Psi>$ contributions to quark self-energies, it would be useful to examine the gauge parameter dependence of such self-energies in a theory where color confinement does not occur, such as standard $SU(2) \times U(1)$ electroweak physics considered in isolation from QCD.

CHAPTER THREE

$<\overline{\Psi}\Psi>$ CONTRIBUTIONS TO ELECTROWEAK QUARK SELF-ENERGIES

3-1) Motivations:

Within spontaneously broken $SU(2)\times U(1)$ electroweak theory, three distinct gauge parameters ξ_A, ξ_W, ξ_Z enter flavor diagonal self-energies, corresponding to distinct degrees of gauge freedom permitted for the photon, the $W^{\pm}$ and the Z intermediate vector bosons. Moreover, the quark mass enters the self-energy nontrivially as well, both through the poles of internal quark lines and through Yukawa couplings to internal scalar field lines of the Higgs sector. The GPI of on shell self-energies in spontaneously broken $SU(2)\times U(1)$ gauge theory is easily shown in a purely perturbative context[32]. One might anticipate a disruption of such on shell gauge parameter cancellations if $SU(2)\times U(1)$ is allowed to couple to a further electroweak symmetry breaking order parameter $<\overline{\Psi}\Psi>$. Indeed a compelling reason for examining the coupling of $<\overline{\Psi}\Psi>$ to $SU(2)\times U(1)$, apart from addressing gauge independence of the on-mass-shell $<\overline{\Psi}\Psi>$ self-energy in a nonconfining context, is our underlying awareness that true standard model physics is the physics of an $SU(3)_c\times SU(2)\times U(1)$ gauge theory, in which the unbroken $SU(3)_c$ sector is sufficiently strong to permit the formation of $<\overline{\Psi}\Psi>$ condensates out of the vacuum. If such condensates exist (from the chiral noninvariance of the QCD vacuum), they will necessarily percolate into the electroweak sector of the standard model.

In this chapter, we examine the gauge parameter independence of on shell quark self-energies in spontaneously broken $SU(2)\times U(1)$ electroweak theory in the presence of an additional D-3 order parameter $<\overline{\Psi}\Psi>$.

3-2) ξ_A dependence of the $<\overline{\Psi}\Psi>$ self-energy contributions:

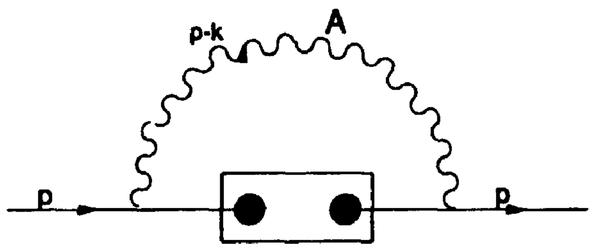


Figure 4: $<\overline{\Psi}\Psi>$ self-energy contribution sensitive to the photon's gauge parameter ξ_A.

We first consider the ξ_A –dependent self-energy contributions (fig.4). Aside from a numerical color factor and a change of coupling constant, the self-energy associated with fig.4 is exactly like (2-18).

$$\sigma(p)^A = \frac{e^2Q^2<\overline{\Psi}\Psi>}{12p^2}\left[(3+\xi_A)-\xi_A\frac{m\hat{p}}{p^2}\right]\Theta(|p^2|-m^2)$$

$$+\frac{e^2Q^2<\overline{\Psi}\Psi>}{12m^2}\left[(3+\xi_A)-\xi_A\frac{\hat{p}}{m}\right]\Theta(m^2-|p^2|), \qquad (3-1)$$

where Q is the fermion electric charge. As in (2-18), the on shell gauge independence of the electromagnetic sector is guaranteed provided the mass shell is identified with m, the OPE mass parameter.

3-2) ξ_Z dependence of the $<\overline{\Psi}\Psi>$ self-energy contribution:

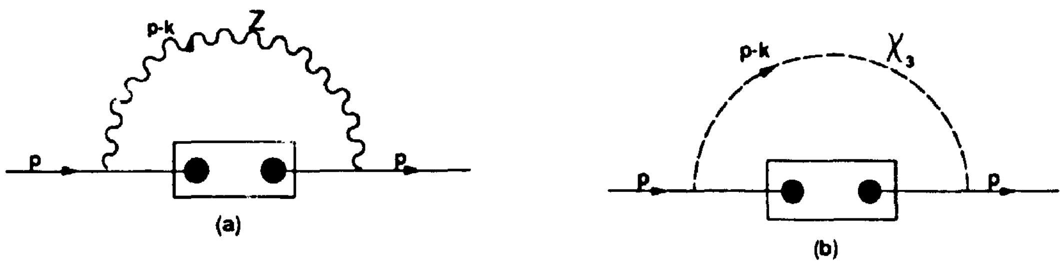


Figure 5: $<\overline{\Psi}\Psi>$ self-energy contributions sensitive to the Z's gauge parameter ξ_Z.

The ξ_Z –dependent contributions to the quark self-energy occur through fig.5, where χ_3 is the scalar partner of the Z gauge boson. The gauge parameter ξ_Z sensitive part of fig. 5a is found using (2-12) to be

$$\sigma^{\xi_Z}_{(5a)}(p) = \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4} \gamma^\mu (a - b\gamma_5) \left(-\frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3} (\hat{k} + m) \delta(k^2 - m^2) \right)$$

$$\gamma^\nu (a - b\gamma_5) \left[\frac{\frac{(p-k)_\mu (p-k)_\nu}{M_z^2}}{(p-k)^2 - \xi_Z M_Z^2} \right] u(p)$$

$$= -\frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3} \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4} \frac{(\hat{p} - \hat{k})(a - b\gamma_5)(\hat{k} + m)(\hat{p} - \hat{k})(a - b\gamma_5)}{M_Z^2((p-k)^2 - \xi_Z M_Z^2)} u(p) \delta(k^2 - m^2),$$

$$(3-2)$$

where a and b are defined in Appendix 1. Using the mass shell condition $\hat{p}u(p) = m u(p)$, we obtain:

$$\sigma^{\xi_Z}_{5a}(p) = -\frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3} \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4} \frac{(m - \hat{k})(a - b\gamma_5)(\hat{k} + m)(a + b\gamma_5)(m - \hat{k})}{M_Z^2((p-k)^2 - \xi_Z M_Z^2)} u(p) \delta(k^2 - m^2)$$

$$= \frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3} \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4} \frac{(2m)(b\gamma_5)(\hat{k} + m)(b\gamma_5)(2m)}{M_Z^2((p-k)^2 - \xi_Z M_Z^2)} u(p) \delta(k^2 - m^2)$$

$$-\frac{4\pi^2 < \overline{\Psi}\Psi >}{3m^3} \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4} \frac{(a^2 + b^2)(m - \hat{k}) + 2mb^2 + 2ab\hat{k}\gamma_5}{M_Z^2((p-k)^2 - \xi_Z M_Z^2)} u(p)(m^2 - k^2) \delta(k^2 - m^2).$$

$$(3-3)$$

The second term in (3-3) vanishes due to the delta function. The contribution from fig. 5b is calculated in a similar manner:

$$\sigma_{5b}^{\xi_Z}(p)=\int\frac{d^4k}{i(2\pi)^4}\left(\frac{-2im^c b\gamma_5}{M_Z}\right)\left(-\frac{4\pi^2<\overline{\Psi}\Psi>}{3m^3}(\not{k}+m)\delta(k^2-m^2)\right)$$

$$\left(\frac{-2im^c b\gamma_5}{M_Z}\right)\left(\frac{1}{\xi_Z M_Z^2-(p-k)^2}\right)$$

$$=-\frac{4\pi^2<\overline{\Psi}\Psi>}{3m^3}\int\frac{d^4k}{i(2\pi)^4}\frac{(2m^c)(b\gamma_5)(\not{k}+m)(b\gamma_5)(2m^c)}{M_Z^2((p-k)^2-\xi_Z M_Z^2)}\delta(k^2-m^2)$$

$$(3-4)$$

The total ξ_Z sensitive self-energy on the fermion mass shell is the sum of (3-3) and (3-4):

$$\sigma_{total}^{\xi_Z}(p)=4\frac{4\pi^2<\overline{\Psi}\Psi>}{3m^3}(m^2-m^{c2})\int\frac{d^4k}{i(2\pi)^4}\frac{(b\gamma_5)(\not{k}+m)(b\gamma_5)}{M_Z^2((p-k)^2-\xi_Z M_Z^2)}\delta(k^2-m^2)$$

$$(3-5)$$

We see that the right hand side of (3-5) vanishes if $m^c=m$.

In other words, the ξ_Z –dependent contributions to the quark self-energy from figs (5a) and (5b) exactly cancel on the quark mass shell, provided that the mass shell retains its identification with the quark mass generated via Yukawa couplings to the spontaneous symmetry breaking parameter $<\Phi>$ [33].

<u>3-3) ξ_W- dependence of the $<\overline{\Psi}\Psi>$ self-energy contribution:</u>

We now consider the ξ_W–dependence of the quark condensate component of quark self-energies. The gauge dependent $<\overline{\Psi}\Psi>$ contribution to the d-quark self-energy (fig.6) is found to be given by:

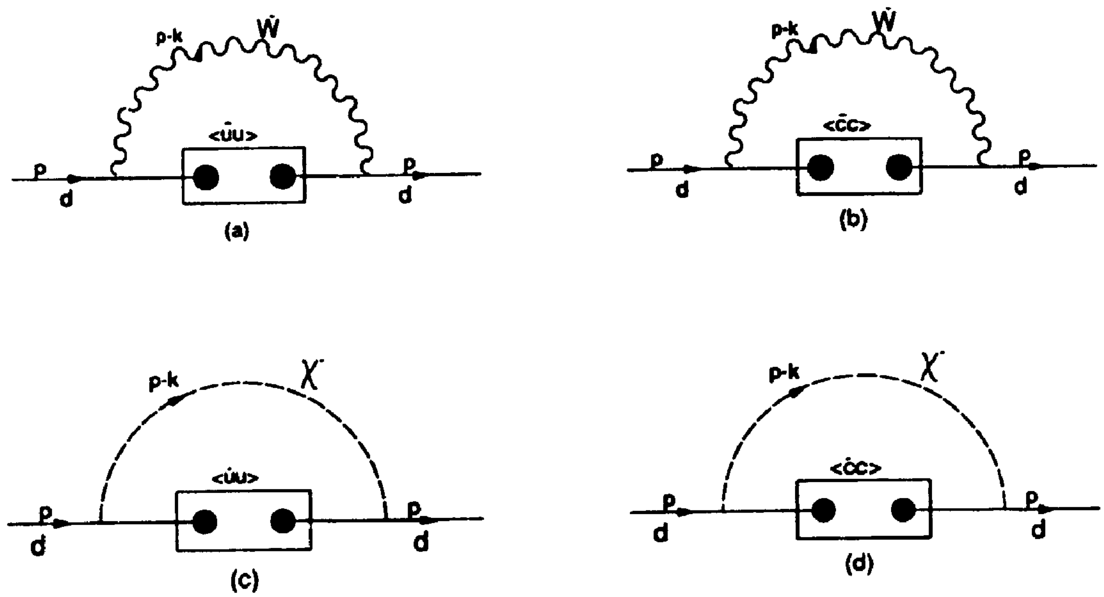


Figure 6: $<\overline{\Psi}\Psi>$ self-energy contributions sensitive to the W's gauge parameter ξ_W.

$$\sigma_{6a}^{\xi_W}(p) = \bar{d}(p)\int \frac{d^4k}{i(2\pi)^4}\left(\frac{g\cos\theta_c}{2\sqrt{2}}\right)\gamma^\mu(1-\gamma_5)\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\hat{k}+m_u)\delta(k^2-m_u^2)\right)\gamma^\nu(1-\gamma_5)$$

$$\left(\frac{g\cos\theta_c}{2\sqrt{2}}\right)\left[\frac{\frac{(p-k)_\mu(p-k)_\nu}{M_W^2}}{(p-k)^2-\xi_W M_W^2}\right]d(p), \qquad (3-6)$$

$$\sigma_{6b}^{\xi_W}(p)=\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\left(\frac{-g\sin\theta_c}{2\sqrt{2}}\right)\gamma^\mu(1-\gamma_5)\left(-\frac{4\pi^2<\bar{c}c>}{3m_c^3}(\hat{k}+m_c)\delta(k^2-m_c^2)\right)\gamma^\nu(1-\gamma_5)$$

$$\left(\frac{-g\sin\theta_c}{2\sqrt{2}}\right)\left[\frac{\frac{(p-k)_\mu(p-k)_\nu}{M_W^2}}{(p-k)^2-\xi_W M_W^2}\right]d(p), \tag{3-7}$$

$$\sigma_{6c}^{\xi_W}(p)=\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\left(-i\frac{g\cos\theta_c}{2\sqrt{2}M_W}\{(m_u-m_d)+(m_u+m_d)\gamma_5\}\right)\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\hat{k}+m_u)\delta(k^2-m_u^2)\right)$$

$$\left(-i\frac{g\cos\theta_c}{2\sqrt{2}M_W}\{(m_d-m_u)+(m_d+m_u)\gamma_5\}\right)\left[\frac{1}{\xi_W M_W^2-(p-k)^2}\right]d(p),$$

(3 – 8)

$$\sigma_{6d}^{\xi_W}(p)=\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\left(i\frac{g\sin\theta_c}{2\sqrt{2}M_W}\{(m_c-m_d)+(m_c+m_d)\gamma_5\}\right)\left(-\frac{4\pi^2<\bar{c}c>}{3m_c^3}(\hat{k}+m_c)\delta(k^2-m_c^2)\right)$$

$$\left(i\frac{g\sin\theta_c}{2\sqrt{2}M_W}\{(m_d-m_c)+(m_d+m_c)\gamma_5\}\right)\left[\frac{1}{\xi_W M_W^2-(p-k)^2}\right]d(p).$$

(3 – 9)

χ^- is the scalar partner of the W gauge boson and θ_c is the Cabibbo mixing angle. In (3-8) and (3-9) no distinction has been made between the masses entering the Yukawa couplings and the corresponding mass shell values for m_u and m_c entering via (2-12). In the last section we found that these masses necessarily agree if $\sigma(p)$ is independent of ξ_Z on the mass shell. Moreover, it is clear from Glashow-Iliopoulos-Maiani (GIM) symmetry[34] that if $\sigma_{6a}^{\xi_W}+\sigma_{6c}^{\xi_W}=0$ on the d-quark mass shell, then $\sigma_{6b}^{\xi_W}+\sigma_{6d}^{\xi_W}=0$ will also vanish on the d-quark mass shell, and on shell diagonal self-energies will be independent of ξ_W for arbitrary values of K-M mixing. Indeed, due to the difference in magnitude between $<\bar{u}u>$ and $<\bar{c}c>$, $\sigma_{6a}^{\xi_W} + \sigma_{6c}^{\xi_W}$ and $\sigma_{6b}^{\xi_W} + \sigma_{6d}^{\xi_W}$ *must separately vanish* for on shell GPI[32].

To obtain this result, let us rewrite (3-6) using the mass shell condition $\hat{p}d(p) = m_d d(p)$:

$$\sigma_{6a}^{\xi_W}(p) = \frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}(\hat{p}-\hat{k})(1-\gamma_5)\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\hat{k}+m_u)\delta(k^2-m_u^2)\right)$$

$$(\hat{p}-\hat{k})(1-\gamma_5)\left[\frac{1}{(p-k)^2-\xi_W M_W^2}\right]d(p)$$

$$= \frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}(m_d-\hat{k})(1-\gamma_5)\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\hat{k}+m_u)\delta(k^2-m_u^2)\right)$$

$$(1+\gamma_5)(m_d-\hat{k})\left[\frac{1}{(p-k)^2-\xi_W M_W^2}\right]d(p)$$

$$= \frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}(m_d-m_u+m_u-\hat{k}-(m_d+m_u-m_u-\hat{k})\gamma_5)$$

$$\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\hat{k}+m_u)\delta(k^2-m_u^2)\right)(m_d-m_u+m_u-\hat{k}$$

$$+\gamma_5(m_d+m_u-m_u-\not{k}))\left[\frac{1}{(p-k)^2-\xi_W M_W^2}\right]d(p)$$

$$=-\frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}(m_u-m_d+(m_d+m_u)\gamma_5)$$

$$\left(-\frac{4\pi^2<\bar{u}u>}{3m_u^3}(\not{k}+m_u)\delta(k^2-m_u^2)\right)(m_d-m_u+\gamma_5(m_d+m_u))\left[\frac{1}{(p-k)^2-\xi_W M_W^2}\right]d(p)$$

$$-\frac{\pi^2<\bar{u}u>g^2\cos^2\theta_c}{3m_u^3M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\frac{[(1+\gamma_5)\not{k}-m_d]\,(k^2-m_u^2)\delta(k^2-m_u^2)}{(p-k)^2-\xi_W M_W^2}d(p)$$

$$(3-10)$$

The second term in the last equality of (3-10) vanishes from the vanishing of $(k^2-m^2)\delta(k^2-m^2)$. The first term in the last equality is equal in magnitude but opposite in sign to $\sigma_{6c}^{\xi_W}$ (3-8), in which case

$$\sigma_{5a}^{\xi_W}(p)+\sigma_{6c}^{\xi_W}(p)=0 \qquad (3-11)$$

Thus the ξ_A, ξ_Z and ξ_W dependence of the flavor diagonal self-energies vanishes on the Dirac-equation mass shell ($\not{p}u(p)=mu(p)$) in perturbative electroweak theory, consistent with the GPI expected for any S-matrix amplitude between physical initial and final states. In this chapter we have shown that this result is maintained when the D-1 order parameter $<\Phi>$ of standard electroweak theory is augmented by a nonzero value for the D-3 order parameter $<\bar{\Psi}\Psi>$, provided the mass characterizing the operator product expansion of the nonvanishing vacuum expectation value (2-1) coincides with both the Dirac-equation mass shell as well as the fermion mass generated via Yukawa coupling to the D-1 order parameter $<\Phi>$.

There is a reassuring self-consistency in these results, as the dependence of spontaneously broken $SU(2)\times U(1)$ theory on the gauge parameters ξ_A, ξ_Z and ξ_W is highly nontrivial. The results obtained above are a strong indication that the on shell gauge independence of $<\overline{\Psi}\Psi>$ contributions to QCD self-energies, demonstrated in section 2-2, is not an accidental consequence of the unbroken gauge symmetry of QCD. Furthermore, the identification of the mass appearing in (2-1) with the quark propagator pole, that is, the Dirac-equation mass shell in the absence of long distance nonperturbative mechanisms for color confinement, is supported by the on shell GPI obtained above through explicit use of this identification.

Since purely perturbative electroweak contributions to the physical on shell quark self-energies are gauge parameter independent within the framework of spontaneously broken $SU(2)\times U(1)$, it is therefore reasonable to use such gauge parameter independence as a test of the theory's self- consistency in the presence of additional order parameter contributions *external* to perturbative $SU(2)\times U(1)$ theory, in particular the quark-antiquark condensates in the physical chiral noninvariant vacuum that presumably arise when the strong $SU(3)_c$ color interactions are taken into account. The GPI in the presence of such a condensate is nontrivial, since a nonzero external value for $<\overline{\Psi}\Psi>$ (analogous to a nonzero external value for $<\Phi>$) explicitly breaks $SU(2)\times U(1)$, respecting only its $U(1)_{EM}$ subgroup.

It is not surprising then that GPI of the on shell self-energy contribution involving a photon continues being trivially satisfied in the presence of a nonzero $<\overline{\Psi}\Psi>$ condensate. By contrast, we see that the GPI of contributions involving a Z or W entails a further self-consistency condition on the mass entering the operator product expansion of normal ordered quark fields (2-1), namely that this operator product mass not only coincide with the fermion mass shell, but also with the corresponding mass generated via Yukawa couplings to the scalar vacuum expectation value $<\Phi>$. This result can be understood to mean that the fermion masses characterizing insertions linear in $<\overline{\Psi}\Psi>$ will not clash with the Ward identities of spontaneously broken $SU(2)\times U(1)$ symmetry provided these fermion masses are in strict correspondence with the masses generated through gauge invariant couplings to the Higgs multiplet responsible for spontaneous symmetry breaking.

On the other hand, from (2-21) we know that there are additional dynamical components to the quark mass due to QCD vacuum condensates. Therefore, to take into account these further chiral symmetry breakings in the electroweak calculations, we may have to correct the relevant Green's functions according to $SU(2)\times U(1)$ Ward identities. The next chapter is devoted to investigating this subject.

CHAPTER FOUR

EXTERNAL SELF-ENERGY CONTRIBUTIONS TO ELECTROWEAK VERTICES

4-1) Quark self-energies external to $SU(2)_L \times U(1)$:

In calculating any electroweak process involving hadrons, it is important to note that the underlying field theory acting on basis quarks is not just $SU(2)_L \times U(1)$ but the full $SU(3)_c \times SU(2)_L \times U(1)$ gauge theory of the standard model. Moreover, the full standard model vacuum necessarily permits perturbative $SU(2)_L \times U(1)$ interactions to couple directly to nonperturbative vacuum condensates. Such nonperturbative contributions to quark two point functions, which appear to be responsible for constituent quark masses (2-19), certainly occur, but are not easily incorporated into electroweak calculations.

In this chapter, we suppose the quark two point function acquires a mass contribution $\sigma(p^2)$ from sources external to perturbative electroweak theory. Terms proportional to $\hat{p}$ $(\equiv \gamma.p)$ in σ are assumed to have already been absorbed in wavefunction renormalization factors (Z_2). Consequently, we will regard σ as a purely Dirac scalar contribution to the fermion propagator:

$$S(p) = (m^c - \hat{p} - \sigma(p^2))^{-1}. \qquad (4-1)$$

For example, in the limit of lagrangian chiral symmetry ($m^c = 0$), the D-3 QCD vacuum condensate $<\overline{\Psi}\Psi>$ yields a self-energy mass contribution (2-19):

$$\sigma(p^2) = \frac{g_s^2 < \overline{\Psi}\Psi > (3+\xi_g)}{9p^2 - \frac{\xi_g m g_s^2 < \overline{\Psi}\Psi >}{p^2}}. \tag{4-2}$$

We will consider $\sigma(p^2)$ to be an *arbitrary* nonperturbative external contribution to the Dirac scalar portion of the quark self-energy. This contribution is not expected to respect the $SU(2)_L \times U(1)$ symmetry of the electroweak lagrangian, as the QCD order parameter $< \overline{\Psi}\Psi >$ is not invariant under $SU(2)_L$ symmetry transformation. Consequently, such external self-energy contributions within electroweak field theoretical processes are expected to lead to calculational inconsistencies. Indeed it was shown in the previous chapter that on-shell electroweak-mediated self-energies in the presence of a quark condensing vacuum, such as that of QCD, exhibit explicit dependence on electroweak gauge parameters unless the quark mass m is equal to lagrangian mass m^c. This would imply that the location of the quark propagator pole is completely insensitive to the $< \overline{\Psi}\Psi >$ condensate, a result inconsistent with having constituent quark masses arise from the chiral noninvariance of the QCD vacuum. Since on shell Feynman amplitudes must be gauge parameter independent for those amplitudes to be meaningful, we can only conclude that a naive application of electroweak Feynman rules is incorrect in the presence of such externally generated quark self-energies.

In the sections which follow, we will obtain electroweak 3 and 4-point Green's functions in the presence of nonperturbative fermion 2-point function contributions external to $SU(2)_L \times U(1)$ through straightforward use of $SU(2)_L \times U(1)$ Ward identities[35]. These Green's functions reduce to those obtained from conventional electroweak Feynman rules in the limit that externally generated quark self-energy contributions vanish.

4-2) Ward Identities of QED:

For the QED subgroup of electroweak theory, the Ward identities relating 3 and 4-point Green's functions to 2-point fermion Green's functions are obtained by requiring invariance of all renormalized Green's functions under the relevant BRST transformations. For example, to obtain the relationship between the quark-antiquark-photon ($\overline{\Psi} A \Psi$) 3-point function and the externally generated self-energy in (4-1), we begin by requiring BRST invariance of the corresponding 3-point function involving the photon's Fadeev-Popov ghost $[c^A, \overline{c}^A]$ (Appendix 2):

$$0 = \delta^{BRST} < \Psi(x)\bar{c}^A(y)\overline{\Psi}(z) >= -i\lambda eQ < \Psi(x)c^A(x)\bar{c}^A(y)\overline{\Psi}(z) >$$

$$+ (\lambda/\xi_A) < \Psi(x)\partial^\mu A_\mu(y)\overline{\Psi}(z) > + i\lambda eQ < \Psi(x)c^A(z)\bar{c}^A(y)\overline{\Psi}(z) >$$

$$+ \text{ contributions from Z and W sectors}$$

$$= -i\lambda eQ < \Psi(x)\overline{\Psi}(z) >< c^A(x)\bar{c}^A(y) > + (\lambda/\xi_A)\partial_y^\mu < \Psi(x)A_\mu(y)\overline{\Psi}(z) >$$

$$+ i\lambda eQ < \Psi(x)\overline{\Psi}(z) >< c^A(z)\bar{c}^{\,\prime}(y) >$$

$$+ \text{ higher order contributions in e and g.} \qquad (4-3)$$

where it is understood that the Green's functions are all VEV of time ordered products of fields. Note that the parameter λ in (4-3) is a Grassman variable. In obtaining the final line of (4-3), we note that the lowest contributing order in electroweak couplings to (4-3) is insensitive to nonabelian contributions from the embedding of QED into $SU(2)_L \times U(1)$. Application of the D'alembertian operator $\Delta \equiv \partial_t^2 - \nabla^2$ in the y-variable yields

$$(1/\xi_A)\Delta_y\partial_y^\mu < \Psi(x)A_\mu(y)\overline{\Psi}(z) >$$

$$= ieQ[< \Psi(x)\overline{\Psi}(z) > \delta^4(x-y) - < \Psi(x)\overline{\Psi}(z) > \delta^4(z-y)]$$

$$+ \text{ higher order contributions in e and g,} \qquad (4-4)$$

where we have used the photon's ghost property $\Delta_y < c^A(x)\bar{c}^A(y) >= \delta^4(x-y)$. If we define our momentum space Green's functions to be related to coordinate space Green's functions via

$$<\Psi(x)\overline{\Psi}(z)>=\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot x}e^{ip\cdot z}G_{\Psi\overline{\Psi}}(q;p) \qquad (4-5a)$$

$$<\Psi(x)A^{\mu}(y)\overline{\Psi}(z)>=\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p}{(2\pi)^4}e^{-iq\cdot x}e^{ik\cdot y}e^{ip\cdot z}G_{\Psi A\overline{\Psi}}(q;k,p), \qquad (4-5b)$$

where momenta before the semicolon are outgoing and momenta after the semicolon are incoming, we then find from (4-4) that

$$-\frac{ik^2k_{\mu}}{\xi_A}G^{\mu}_{\Psi A\overline{\Psi}}(q;k,p)=ieQ[G_{\Psi\overline{\Psi}}(q-k;p)-G_{\Psi\overline{\Psi}}(q;p+k)]. \qquad (4-6)$$

If we define truncated momentum space Green's functions via

$$G^{\mu}_{\Psi A\overline{\Psi}}(q;k,p)=S(q)\Delta^{\mu\nu}_{A}(k)\Gamma^{\nu}_{\overline{\Psi}A\Psi}(q;k,p)S(p)\delta^4(q-k-p) \qquad (4-7a)$$

$$G_{\Psi\overline{\Psi}}(q-k;p)=S(p)\delta^4(q-k-p), \qquad (4-7b)$$

and make use of the Ward identity for the full photon propagator

$$k_{\mu}\Delta^{\mu\nu}_{A}(k)=\frac{\xi_A k^{\nu}}{k^2}, \qquad (4-8)$$

a result which can be derived from the BRST invariance of $<A_{\mu}(x)\overline{c}^{A}(y)>$, we then obtain the well known relation between two and three point functions that is upheld to all orders of QED considered in isolation:

$$k_{\mu}\Gamma^{\mu}_{\overline{\Psi}A\Psi}(q;k,p)=-eQ[S^{-1}(q)-S^{-1}(p)]. \qquad (4-9)$$

Equation (4-9) demonstrates how an external momentum dependent contribution to the propagator (4-1) necessarily alters the off shell coupling of fermions to photons. The contribution (4-9), of course, vanishes with vanishing photon momentum, consistent with the definition of electric charge.

Now we derive the QED Ward identity relating the $\Psi AA\overline{\Psi}$ 4-point function to the 2 and 3-point functions of (4-5) by considering the f ˙˙owing BRST variation:

$$
\begin{aligned}
0 &= \delta^{BRST} < \Psi(x) A_\mu(y) \bar{c}^A(w) \overline{\Psi}(z) > \\
&= -i\lambda eQ < \Psi(x) A_\mu(y) c^A(x) \bar{c}^A(w) \overline{\Psi}(z) > - \lambda < \Psi(x) [\partial_\mu c^A(y)] \bar{c}^A(w) \overline{\Psi}(z) > \\
&+ (\lambda/\xi_A) < \Psi(x) A_\mu(y) [\partial^\nu A_\nu(w)] \overline{\Psi}(z) > + i\lambda eQ < \Psi(x) A_\mu(y) c^A(z) \bar{c}^A(w) \overline{\Psi}(z) > \\
&+ \text{contributions from Z and W sectors} \\
&= -i\lambda eQ < \Psi(x) A_\mu(y) \overline{\Psi}(z) >< c^A(x) \bar{c}^A(w) > - \lambda < \Psi(x) \overline{\Psi}(z) > (\partial_y)_\mu < c^A(y) \bar{c}^A(w) > \\
&+ (\lambda/\xi_A) \partial^\nu_w < \Psi(x) A_\mu(y) A_\nu(w) \overline{\Psi}(z) > + i\lambda eQ < \Psi(x) A_\mu(y) \overline{\Psi}(z) >< c^A(z) \bar{c}^A(w) > \\
&+ \text{higher order contributions in e and g.} \qquad (4-10)
\end{aligned}
$$

If we ignore the higher order contributions to (4-10) and apply a D'alembertian operator in the variable w, we obtain

$$\frac{1}{\xi_A}\Delta_w\partial_w^\nu < \Psi(x)A_\mu(y)A_\nu(w)\overline{\Psi}(z) >$$

$$= ieQ < \Psi(x)A_\mu(y)\overline{\Psi}(z) > \delta^4(x - w)$$

$$+ (\partial_y)_\mu [< \Psi(x)\overline{\Psi}(z) > \delta^4(y - w)]$$

$$- ieQ < \Psi(x)A_\mu(y)\overline{\Psi}(z) > \delta^4(z - w). \qquad (4-11)$$

The 4-point function in (4-11) has a connected and a disconnected piece:

$$< \Psi(x)A_\mu(y)A_\nu(w)\overline{\Psi}(z) > = < \Psi(x)A_\mu(y)A_\nu(w)\overline{\Psi}(z) >_c$$

$$+ < \Psi(x)\overline{\Psi}(z) >< A_\mu(y)A_\nu(w) > . \qquad (4-12)$$

(4-11) can be used to generate a momentum space Ward identity through use of (4-7), (4-12), and the momentum-space connected Green's functions:

$$< A^\mu(y)A^\nu(w) > = \int\frac{d^4p}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{-ip.y}e^{iq.w}G^{\mu\nu}_{AA}(p;q)$$

$$G^{\mu\nu}_{AA}(p;q) = \Delta^{\mu\nu}_A(q)\delta^4(p - q)$$

$$< \Psi(x)A^\mu(y)A^\nu(w)\overline{\Psi}(z) >_c$$

$$= \int\frac{d^4l}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\int\frac{d^4p}{(2\pi)^4}e^{-il.x}e^{ik.y}e^{iq.w}e^{ip.z}G^{\mu\nu}_{\Psi AA\overline{\Psi}}(l;k,q,p).$$

$$(4-13)$$

We then find from (4-11) that

$$-\frac{iq^2q_\nu}{\xi_A}\left[G^{\mu\nu}_{\Psi AA\bar\Psi}(l;k,q,p)+G_{\Psi\bar\Psi}(l;p)G^{\mu\nu}_{AA}(-k;q)\right]$$

$$=ieQ\left[G^{\mu}_{\Psi A\bar\Psi}(l-q;k,p)-G^{\mu}_{\Psi A\bar\Psi}(l;k,p+q)\right]+ik^\mu G_{\Psi\bar\Psi}(l;p)\delta^4(k+p).$$

$$(4-14)$$

We define the following additional truncated momentum space Green's functions to supplement those of (4-7):

$$G^{\mu\nu}_{\Psi AA\bar\Psi}(l;k,q,p)=S(l)\Delta^{\mu\rho}_A(k)\Delta^{\nu\sigma}_A(q)\Gamma^{\tau\eta}_{\bar\Psi AA\Psi}(l;k,q,p)g_{\rho\tau}g_{\sigma\eta}S(p)\delta^4(l-k-q-p)$$

$$G^{\mu\nu}_{AA}(-k;q)=\Delta^{\mu\nu}_a(q)\delta^4(q+k). \qquad (4-15)$$

Upon substitution of (4-8) and (4-15) into (4-14), we obtain the following Ward identity (l=k+q+p):

$$q_\sigma\Gamma^{\tau\sigma}_{\bar\Psi AA\Psi}(l;k,q,p)$$

$$=-eQ[S^{-1}(l)S(l-q)\Gamma^{\tau}_{\bar\Psi A\Psi}(l-q;k,p)$$

$$-\Gamma^{\tau}_{\bar\Psi A\Psi}(l;k,p+q)S(p+q)S^{-1}(p)]. \qquad (4-16)$$

In (4-14), we may use (4-9) to replace $S^{-1}(l)$ and $S^{-1}(p)$ with the following expressions:

$$-eQS^{-1}(l)=q_\nu\Gamma^{\nu}_{\bar\Psi A\Psi}(l;q,l-q)-eQS^{-1}(l-q),$$

$$eQS^{-1}(p)=eQS^{-1}(p+q)+q_\nu\Gamma^{\nu}_{\bar\Psi A\Psi}(p+q;q,p), \qquad (4-17)$$

so as to obtain

$$q_\sigma \Gamma^{\tau\sigma}_{\bar{\Psi}AA\Psi}(l;k,q,p)$$

$$= q_\sigma\{\Gamma^{\sigma}_{\bar{\Psi}A\Psi}(l;q,l-q)S(l-q)\Gamma^{\tau}_{\bar{\Psi}A\Psi}(l-q;k,p)$$

$$+\Gamma^{\tau}_{\bar{\Psi}A\Psi}(l;k,p+q)S(p+q)\Gamma^{\sigma}_{\bar{\Psi}A\Psi}(p+q;q,p)\}$$

$$+eQ\left[\Gamma^{\tau}_{\bar{\Psi}A\Psi}(l;k,p+q)-\Gamma^{\tau}_{\bar{\Psi}A\Psi}(l-q;k,p)\right]. \qquad (4-18)$$

This last identity reveals the structure of the $\bar{\Psi}AA\Psi$ 4-point function in the presence of external contributions to the quark self-energy. The curly bracketed term on the right hand side of (4-18) corresponds to the 1PR (one-point reducible contributions) one would obtain in field theory from appropriately dressed three-point vertices and two point propagators arising from the QED lagrangian. The final square bracketed term on the right hand side of (4-18) can easily be shown to vanish in the absence of external momentum dependent contributions to the fermion self-energy. However, in the presence of an external momentum dependent contribution to the self-energy as proposed in (4-1), the final term in (4-18) corresponds to a distinct 1PI (one-point irreducible) contribution to the truncated four-point Green's function, as indicated schematically in fig. 7. This additional contribution is essential for gauge parameter independence of the on shell quark self-energy, as will be demonstrated later.

4-3) Weak neutral current Ward identities:

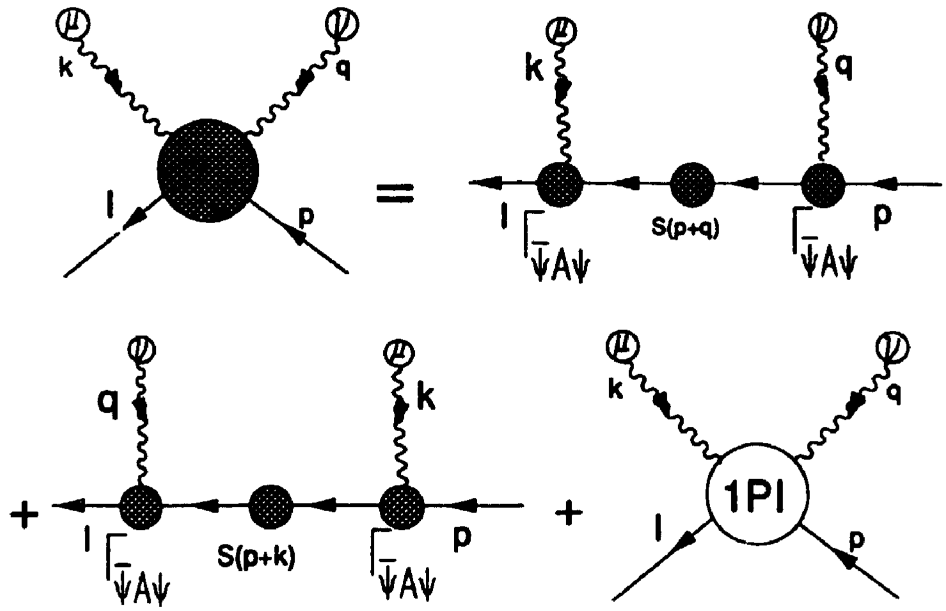


Figure 7: $\overline{\Psi}AA\Psi$ 4-point function in the presence of external contributions to the quark self-energy.

In this section, we derive in detail equations analogous to (4-9) and (4-18) involving Z sector three and four point functions. In deriving Ward identities for Z coupled Green's functions , one must take into account nonvanishing Yukawa coupling to the Higgs sector. Consequently both the physical Higgs field (Φ) and unphysical scalar partner (χ_3) to the Z will appear in relevant Ward identities. As in the previous section, we begin by requiring BRST invariance of an appropriately chosen 3-point function (Appendix 2):

$$0 = \delta^{BRST} < \Psi(x)\bar{c}^{Z}(y)\overline{\Psi}(z) >$$

$$= -i\lambda(a - b\gamma_5) < \Psi(x)c^{Z}(x)\bar{c}^{Z}(y)\overline{\Psi}(z) >$$

$$+ \lambda < \Psi(x)\left(\frac{1}{\xi_Z}\partial^{\mu}Z_{\mu}(y) + M_Z\chi_3(y)\right)\overline{\Psi}(z) >$$

$$+ i\lambda < \Psi(x)c^{Z}(z)\bar{c}^{Z}(y)\overline{\Psi}(z) > (a + b\gamma_5), \qquad (4-19)$$

where ξ_Z is the ('t Hooft-Feynman) gauge parameter of the Z, and where a and b characterize the tree level $\overline{\Psi}Z\Psi$ vertex (Appendix 1). Upon application of the operator $(\Delta_y + \xi_Z M_Z^2)$ to both sides of (4-19) and utilization of the ghost field's 2-point Green's function equation, we find (to lowest order in electroweak couplings) that

$$\frac{1}{\xi_Z}\partial_y^{\mu}(\Delta_y + \xi_Z M_Z^2) < \Psi(x)Z_{\mu}(y)\overline{\Psi}(z) >$$

$$+ M_Z(\Delta_y + \xi_Z M_Z^2) < \Psi(x)\chi_3(y)\overline{\Psi}(z) >$$

$$= -i < \Psi(x)\overline{\Psi}(z) > (a + b\gamma_5)\delta^4(z - y)$$

$$+ i(a - b\gamma_5) < \Psi(x)\overline{\Psi}(z) > \delta^4(x - y). \qquad (4-20)$$

We define momentum space 2 and 3-point Green's functions as in (4-5) in order to obtain the relationship

$$\frac{1}{\xi_Z}(ik_\mu)(-k^2+\xi_Z M_Z^2)G^\mu_{\Psi Z\overline{\Psi}}(q;k,p)$$

$$+M_Z(-k^2+\xi_Z M_Z^2)G_{\Psi\chi_3\overline{\Psi}}(q;k,p)$$

$$=i(a-b\gamma_5)G_{\Psi\overline{\Psi}}(q-k;p)-iG_{\Psi\overline{\Psi}}(q;p+k)(a+b\gamma_5). \quad (4-21)$$

To express (4-21) in terms of truncated Green's functions, we utilize (4-7), and

$$G_{\Psi\chi_3\overline{\Psi}}(q;k,p)=S(q)\Delta_{\chi_3}(k)\Gamma_{\overline{\Psi}\chi_3\Psi}(q;k,p)S(p)\delta^4(q-k-p), \qquad (4-22a)$$

$$G^\mu_{\Psi Z\overline{\Psi}}(q;k,p)=S(q)\Delta_Z^{\mu\nu}(k)g_{\nu\tau}\Gamma^\tau_{\overline{\Psi}Z\Psi}(q;k,p)S(p)\delta^4(q-k-p), \qquad (4-22b)$$

as well as Z and χ_3 propagator identities (following from BRST invariance of $<Z_\mu(x)\overline{c}^Z(y)>, <\chi_3(x)\overline{c}^Z(y)>$):

$$(-k^2+\xi_Z M_Z^2)\Delta_{\chi_3}(k)=1, \qquad (4-23a)$$

$$k_\mu\Delta_Z^{\mu\nu}(k)=\frac{\xi_Z k^\nu}{k^2-\xi_Z M_Z^2}. \qquad (4-23b)$$

We then find that

$$-ik_\nu\Gamma^\nu_{\overline{\Psi}Z\Psi}(q;k,p)+M_Z\Gamma_{\overline{\Psi}\chi_3\Psi}(q;k,p)=iS^{-1}(q)(a-b\gamma_5)-i(a+b\gamma_5)S^{-1}(p).$$

$$(4-24)$$

Equation (4-24) is easily seen to be satisfied by the tree level vertices (Feynman rules of Appendix 1) of $SU(2)\times U(1)$. We see from (4-24), as in (4-9), that external contributions to the fermion propagator (4-1) necessarily affect $\overline{\Psi}Z\Psi$ and $\overline{\Psi}\chi_3\Psi$ three point Green's functions.

To obtain Z sector analogues of (4-18) relating 4 point Green's functions to 2 and 3 point Green's functions, we follow the procedures delineated prior to (4-18) by using BRST invariance of a 4 point function:

$$
\begin{aligned}
0 &= \delta^{BRST} < \Psi(x) Z_\mu(y) \bar{c}^Z(w) \overline{\Psi}(z) > \\
&= -i\lambda(a - b\gamma_5) < \Psi(x) Z_\mu(y) c^Z(x) \bar{c}^Z(w) \overline{\Psi}(z) > \\
&- \lambda < \Psi(x) (\partial_\mu c^Z(y)) \bar{c}^Z(w) \overline{\Psi}(z) > \\
&+ \lambda < \Psi(x) Z_\mu(y) \left(\frac{1}{\xi_Z} \partial^\nu Z_\nu(w) + M_Z \chi_3(w) \right) \overline{\Psi}(z) > \\
&+ i\lambda < \Psi(x) Z_\mu(y) c^Z(z) \bar{c}^Z(w) \overline{\Psi}(z) > (a + b\gamma_5). \qquad (4-25)
\end{aligned}
$$

This leads to the configuration space identity

$$
\begin{aligned}
&(\Delta_w + \xi_Z M_Z^2) \{ \frac{1}{\xi_Z} \partial_w^\nu \left(< \Psi(x) Z_\mu(y) Z_\nu(w) \overline{\Psi}(z) >_c + < \Psi(x) \overline{\Psi}(z) >< Z_\mu(y) Z_\nu(w) > \right) \\
&- \partial_y^\mu < \Psi(x) c^Z(y) \bar{c}^Z(w) \overline{\Psi}(z) > + M_Z < \Psi(x) Z_\mu(y) \chi_3(w) \overline{\Psi}(z) > \} \\
&= i(a - b\gamma_5) < \Psi(x) Z_\mu(y) \overline{\Psi}(z) > \delta^4(x - w) \\
&- i < \Psi(x) Z_\mu(y) \overline{\Psi}(z) > (a + b\gamma_5) \delta^4(z - w). \qquad (4-26)
\end{aligned}
$$

In terms of truncated momentum space Green's functions, one obtains the Ward identity

$$-iq_\nu\Gamma^{\nu\mu}_{\Psi ZZ\Psi}(l;k,q,p)+M_Z\Gamma^{\mu}_{\Psi Z\chi_3\Psi}(l;k,q,p)$$

$$=iS^{-1}(l)(a-b\gamma_5)S(p+k)\Gamma^{\mu}_{\Psi Z\Psi}(p+k;k,p)$$

$$-i\Gamma^{\mu}_{\Psi Z\Psi}(l;k.p+q)S(p+q)(a+b\gamma_5)S^{-1}(p)$$

$$-\frac{ik^\mu}{\xi_Z}\Gamma_{\Psi\bar{c}c\Psi}(l;k,q,p) \qquad (4-27)$$

through appropriate use of (4-23). Similarly, one obtains from

$$0=\delta^{BRST}<\Psi(x)\chi_3(w)\bar{c}^Z(y)\overline{\Psi}(z)>$$

$$=-i\lambda(a-b\gamma_5)<\Psi(x)\chi_3(w)c^Z(x)\bar{c}^Z(y)\overline{\Psi}(z)>$$

$$-2b\lambda<\Psi(x)(<\Phi>+\Phi(w))c^Z(w)\bar{c}^Z(y)\overline{\Psi}(z)>$$

$$+\lambda<\Psi(x)\chi_3(w)\left(\frac{1}{\xi_Z}\partial^\mu Z_\mu(y)+M_Z\chi_3(y)\right)\overline{\Psi}(z)>$$

$$+i\lambda<\Psi(x)\chi_3(w)c^Z(z)\bar{c}^Z(y)\overline{\Psi}(z)>(a+b\gamma_5) \qquad (4-28)$$

($\Phi(w)$ is the physical Higgs field, and $<\Phi>$ is the corresponding vacuum expectation value) the momentum space Ward identity (l-q=p+k)

$$
\begin{aligned}
&-iq_\mu \Gamma^\mu_{\bar{\Psi}\chi_3 Z\Psi}(l;k,q,p) + M_Z \Gamma_{\bar{\Psi}\chi_3\chi_3\Psi}(l;k,q,p) \\
&= iS^{-1}(l)(a - b\gamma_5)S(l-q)\Gamma_{\bar{\Psi}\chi_3\Psi}(l-q;k,p) + 2b\frac{\xi_Z M_Z^2 - k^2}{m_\Phi^2 - (k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p) \\
&- i\Gamma_{\bar{\Psi}\chi_3\Psi}(l;k,p+q)S(p+q)(a + b\gamma_5)S^{-1}(p) + M_Z \Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;k,q,p).
\end{aligned}
\tag{4-29}
$$

In obtaining (4-29) we have utilized both the tree level relationship $2b < \Phi > = M_Z$ and the tree level expression for the Φ propagator, as is appropriate for the (lowest nontrivial) order of electroweak coupling to which we are working. Equation (4-27) and (4-29) can be employed to eliminate $\Gamma^\mu_{\bar{\Psi}\chi_3 Z\Psi}$, thereby yielding the relationship

$$q_\nu k_\mu \Gamma^{\mu\nu}_{\bar{\Psi}ZZ\Psi}(l;k,q,p) + M_Z^2 \Gamma_{\bar{\Psi}\chi_3\chi_3\Psi}(l;q,k,p)$$

$$= -S^{-1}(l)(a-b\gamma_5)S(p+k)k_\mu \Gamma^{\mu}_{\bar{\Psi}Z\Psi}(p+k;k,p)$$

$$+ k_\mu \Gamma^{\mu}_{\bar{\Psi}Z\Psi}(l;k,p+q)S(p+q)(a+b\gamma_5)S^{-1}(p)$$

$$+ \frac{k^2}{\xi_Z}\Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;k,q,p)$$

$$+ iS^{-1}(l)(a-b\gamma_5)S(l-k)M_Z \Gamma_{\bar{\Psi}\chi_3\Psi}(l-k;q,p)$$

$$+ 2bM_Z \frac{\xi_Z M_Z^2 - q^2}{m_\Phi - (k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p)$$

$$- iM_Z \Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k)S(p+k)(a+b\gamma_5)S^{-1}(p)$$

$$+ M_Z^2 \Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;q,k,p). \tag{4-30}$$

We note from Bose symmetry that

$$q_\nu k_\mu \Gamma^{\mu\nu}_{\bar{\Psi}ZZ\Psi}(l;k,q,p) = k_\nu q_\mu \Gamma^{\mu\nu}_{\bar{\Psi}ZZ\Psi}(l;q,k,p), \tag{4-31a}$$

$$\Gamma_{\bar{\Psi}\chi_3\chi_3\Psi}(l;q,k,p) = \Gamma_{\bar{\Psi}\chi_3\chi_3\Psi}(l;k,q,p), \tag{4-31b}$$

Moreover, careful consideration of 4 point $\bar{\Psi}\bar{c}c\Psi$ Green's function shows it to be invariant under exchange of inbound ghost momenta:

$$\Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;k,q,p) = \Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;q,k,p). \tag{4-31c}$$

This last property directly follows from the momentum exchange symmetry of the $\bar{c}\Phi c$ vertex, as can be verified by explicit construction of the (uncorrected) $\bar{\Psi}\bar{c}c\Psi$ 4point function from electroweak vertices. If we subtract from (4-30) the version of (4-30) we would have upon exchanging k and q, we can make use of (4-31) and a judicious rearrangement of terms to obtain the identity

$$
\begin{aligned}
0 = & -S^{-1}(l)(a-b\gamma_5)S(p+k)\left[k_\mu\Gamma^\mu_{\bar{\Psi}Z\Psi}(p+k;k,p)+iM_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(p+k;k,p)\right] \\
& +\left[k_\mu\Gamma^\mu_{\bar{\Psi}Z\Psi}(l;k,p+q)+iM_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(l;k,p+q)\right]S(p+q)(a+b\gamma_5)S^{-1}(p) \\
& +S^{-1}(l)(a-b\gamma_5)S(p+q)\left[iM_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(p+q;q,p)+q_\mu\Gamma^\mu_{\bar{\Psi}Z\Psi}(p+q;q,p)\right] \\
& -\left[iM_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k)+q_\mu\Gamma^\mu_{\bar{\Psi}Z\Psi}(l;q,p+k)\right]S(p+k)(a+b\gamma_5)S^{-1}(p) \\
& +\frac{k^2-q^2}{\xi_Z}\Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;k,q,p)+2bM_Z\frac{k^2-q^2}{m_\Phi^2-(q+k)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p). \qquad (4-32)
\end{aligned}
$$

We now apply (4-24) to all square bracketed terms in (4-32). The only surviving terms in (4-32) then yield the relationship depicted graphically in fig. 8:

$$
\Gamma_{\bar{\Psi}\bar{c}c\Psi}(l;k,q,p) = -\frac{2b\xi_z M_Z}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p). \qquad (4-33)
$$

It must be emphasized that (4-33) is not a self-evident result in the presence of external self-energy contributions; the absence of a 1PI contribution analogous to that of (4-18) follows from the complete cancellation of all 2 point function contributions within (4-32). The result (4-33), however, demonstrates the anticipated insensitivity of 3 and 4 point functions not involving fermions to external fermion contributions, a property we have found

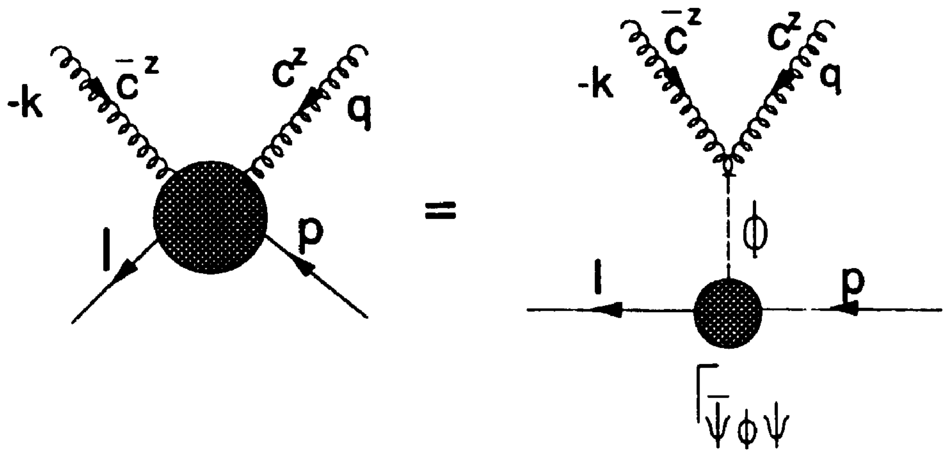


Figure 8: The $\bar{\Psi}\bar{c}^Z c^Z \Psi$ truncated momentum space Green's function to lowest contributing order in electroweak coupling.

consistently to be upheld. For example, we see from (4-33) that, to lowest contributing order in electroweak couplings, the tree level $\bar{c}\Phi c$ vertex $(-2b\xi_Z M_Z)$ is impervious to external contributions to the fermion propagator. Such is not necessarily the case for 3 and 4 point functions that do involve fermions; sensitivity of such vertices to external contributions to S^{-1} [e.g. σ in (4-1)] is evident in (4-24).

We now substitute (4-33) into both (4-27) and (4-29). We also use (4-24) to replace respectively factors of $iS^{-1}(l)(a - b\gamma_5)$ and $-i(a + b\gamma_5)S^{-1}(p)$ common to the right hand sides of both (4-27) and (4-29) with $-iq_\nu\Gamma^\nu_{\bar{\Psi}Z\Psi}(l;q,p+k) + M_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k) + i(a+b\gamma_5)S^{-1}(p+k)$ and

$-iq_\nu\Gamma^{\nu}_{\bar{\Psi}Z\Psi}(p+q;q,p)+M_Z\Gamma_{\bar{\Psi}\chi_3\Psi}(p+q;q,p)-iS^{-1}(p+q)(a-b\gamma_5)$. The substitution into (4-27) yields the following result after some algebraic rearrangement:

$$
\begin{aligned}
&-i\left[q_\nu\Gamma^{\mu\nu}_{\bar{\Psi}ZZ\Psi}(l;k,q,p)\right]+M_Z\left\{\Gamma^{\mu}_{\bar{\Psi}Z\chi_3\Psi}(l;k,q,p)\right\}\\
&=-i[q_\nu\Gamma^{\nu}_{\bar{\Psi}Z\Psi}(l;q,p+k)S(p+k)\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(p+k;k,p)\\
&+q_\nu\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(l;k,p+q)S(p+q)\Gamma^{\nu}_{\bar{\Psi}Z\Psi}(p+q;q,p)\\
&+\frac{4bq^\mu M_Z}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p)]\\
&+M_Z\{\Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k)S(p+k)\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(p+k;k,p)\\
&+\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\Psi}\chi_3\Psi}(p+q;q,p)\\
&+\frac{2ib(k^\mu+2q^\mu)}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p)\}\\
&+i(a+b\gamma_5)\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(p+k;k,p)-i\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(l;k,p+q)(a-b\gamma_5). \qquad (4-34)
\end{aligned}
$$

In our arrangement of the right hand side of (4-34), the terms enclosed by square brackets correspond to the one particle reducible contributions (1PR) to the $\bar{\Psi}ZZ\Psi$ Green's function delineated in fig. 9. Similarly, the terms in large curly brackets correspond to the 1PR contributions to the $\bar{\Psi}Z\chi_3\Psi$ Green's function delineated in fig. 10.

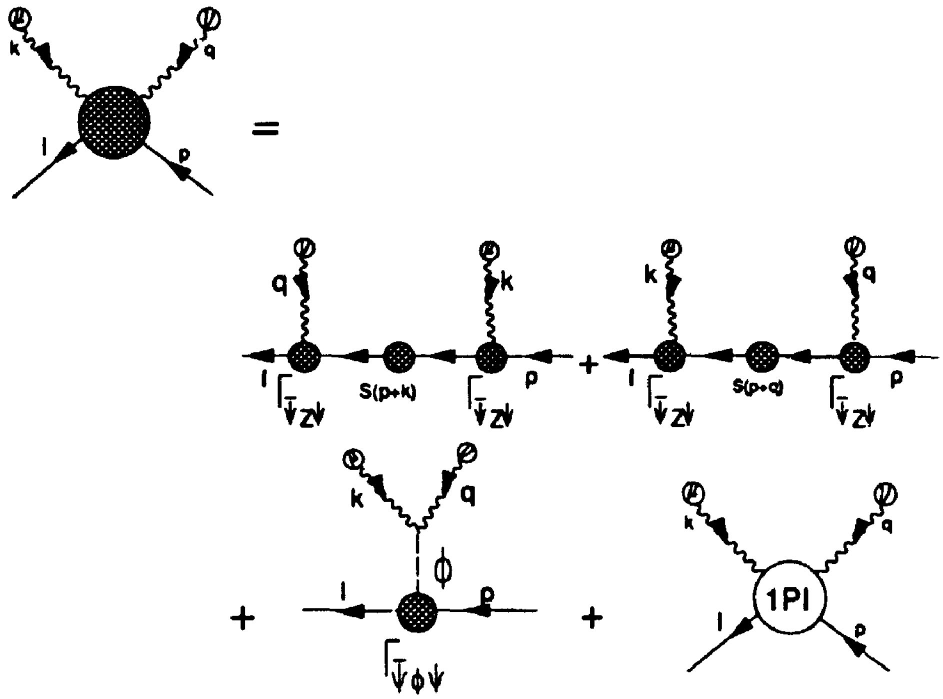


Figure 9: 1PR and 1PI components of the $\overline{\Psi}ZZ\Psi$ truncated Green's function.

The final unbracketed terms in (4-34) vanish at tree level, but become a nonvanishing 1PI contribution in the presence of external contributions to the quark self-energy. By choosing (4-34) and (4-18) to be consistent in the limit $M_Z \rightarrow 0, a \rightarrow eQ, b \rightarrow 0$ $[\overline{\Psi}ZZ\Psi \rightarrow \overline{\Psi}AA\Psi]$, we find that (fig. 9)

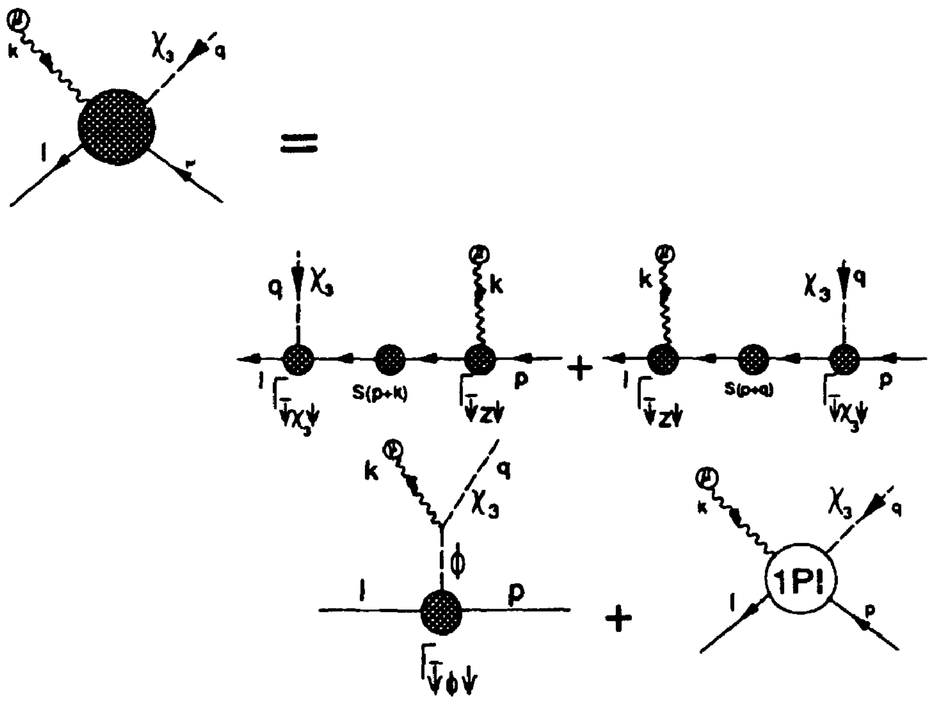


Figure 10: 1PR components of the $\bar\Psi Z\chi_3\Psi$ truncated Green's function.

$$q_\nu \Gamma^{\mu\nu}_{\bar\Psi ZZ\Psi}(l;k,q,p) = [q_\nu \Gamma^{\nu}_{\bar\Psi Z\Psi}(l;q,p+k) S(p+k)\Gamma^{\mu}_{\bar\Psi Z\Psi}(p+k;k,p)$$

$$+ \Gamma^{\mu}_{\bar\Psi Z\Psi}(l;k,p+q) S(p+q) q_\nu \Gamma_{\bar\Psi Z\Psi}(p+q;q,p)$$

$$+ \Gamma_{\bar\Psi\Phi\Psi}(l;q+k,p)\frac{4bM_Z q^\mu}{m_\Phi^2-(q+k)^2}]$$

$$+ \Gamma^{\mu}_{\bar\Psi Z\Psi}(l;k,p+q)(a-b\gamma_5) - (a+b\gamma_5)\Gamma^{\mu}_{\bar\Psi Z\Psi}(p+k;k,p),$$

$$(4-35)$$

in which case the $\bar\Psi Z\chi_3\Psi$ Green's function is purely 1PR (fig. 10):

$$\Gamma^{\mu}_{\bar{\Psi}Z\chi_3\Psi}(l;k,q,p) = \{\Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k)S(p+k)\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(p+k;k,p)$$

$$+\Gamma^{\mu}_{\bar{\Psi}Z\Psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\Psi}\chi_3\Psi}(p+q;q,p)$$

$$+\Gamma_{\bar{\Psi}\Phi\Psi}(l;q+k,p)\frac{2ib(k^{\mu}+2q^{\mu})}{m_{\Phi}^2-(q+k)^2}\}. \qquad (4-36)$$

This choice is also supported by requiring smoothness of Green's functions in the $<\Phi>\to 0$ limit. We note that our decision to include the 1PI contributions proportional to b in the $\bar{\Psi}ZZ\Psi$ Green's function (4-35) involves some arbitrariness; there is sufficient freedom in the equations to allow a different partition of 1PI contributions without spoiling the correspondence between the $\bar{\Psi}ZZ\Psi$ and $\bar{\Psi}AA\Psi$ Green's functions in the $b\to 0$ limit.

The corresponding substitutions of (4-33) and the above described versions of (4-24) into (4-29) yield, after suitable algebraic rearrangement, the following expression:

$$-i\left\{ q_\mu \Gamma^\mu_{\bar{\Psi}\chi_3 Z\Psi}(l;k,q,p)\right\} + M_Z\left[\Gamma_{\bar{\Psi}\chi_3\chi_3\Psi}(l;k,q,p)\right]$$

$$= -i\{q_\nu \Gamma^\nu_{\bar{\Psi}Z\Psi}(l;q,p+k)S(p+k)\Gamma_{\bar{\Psi}\chi_3\Psi}(p+k;k,p)$$

$$+q_\nu \Gamma_{\bar{\Psi}\chi_3\Psi}(l;k,p+q)S(p+q)\Gamma^\nu_{\bar{\Psi}Z\Psi}(p+q;q,p)$$

$$+\frac{2ib(q^2+2k.q)}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p)\}$$

$$+M_Z[\Gamma_{\bar{\Psi}\chi_3\Psi}(l;q,p+k)S(p+k)\Gamma_{\bar{\Psi}\chi_3\Psi}(p+k;k,p)$$

$$+\Gamma_{\bar{\Psi}\chi_3\Psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\Psi}\chi_3\Psi}(p+q;q,p)$$

$$-\frac{2bm_\Phi^2/M_Z}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p)]$$

$$+i(a+b\gamma_5)\Gamma_{\bar{\Psi}\chi_3\Psi}(l-q;k,p)-i\Gamma_{\bar{\Psi}\chi_3\Psi}(l;k,p+q)(a-b\gamma_5)$$

$$+2b\Gamma_{\bar{\Psi}\Phi\Psi}(l;k+q,p). \qquad (4-37)$$

The curly bracketed terms on both sides of (4-37) are equal by (4-36). Note that q is the Z-leg momentum in (4-37) and the χ_3 –leg momentum in (4-36). The terms within square brackets on the right hand side of (4-37) are the 1PR contributions to the $\bar{\Psi}\chi_3\chi_3\Psi$ Green's function delineated in fig. 11.

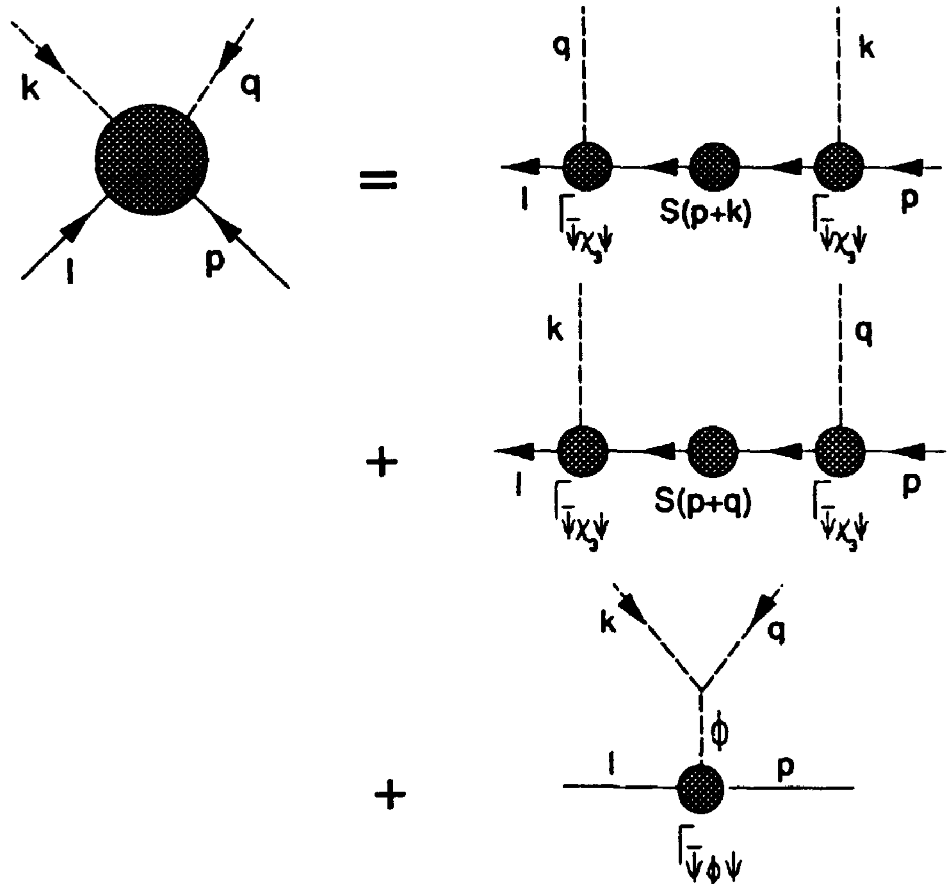


Figure 11: 1PR components of the $\overline{\Psi}\chi_3\chi_3\Psi$ truncated Green's function.

The remaining unbracketed terms on the right hand side of (4-37) separately vanish at tree level. Sufficient freedom remains in our system to permit the maintenance of the tree level relationship between $\overline{\Psi}\Phi\Psi$ and $\overline{\Psi}\chi_3\Psi$ vertices:

$$\Gamma_{\overline{\Psi}\Phi\Psi}(l;k+q,p) = -\frac{i}{2}\left[\gamma_5\Gamma_{\overline{\Psi}\chi_3\Psi}(l-q;k,p) + \Gamma_{\overline{\Psi}\chi_3\Psi}(l;k,p+q)\gamma_5\right], \qquad (4-38)$$

thereby retaining consistency with having Φ and χ_3 generated from the neutral component of the original scalar field doublet. Application of (4-38) to the unbracketed terms in (4-37) yields cancellation of all such terms with coefficient "b". The remaining unbracketed "a" terms correspond to a possible 1PI contribution to the $\overline{\Psi}\chi_3\chi_3\Psi$ Green's function:

$$
\begin{aligned}
&\Gamma_{\overline{\Psi}\chi_3\chi_3\Psi}(l;k,q,p) \\
&= \Gamma_{\overline{\Psi}\chi_3\Psi}(l;q,p+k)S(p+k)\Gamma_{\overline{\Psi}\chi_3\Psi}(p+k;k,p) \\
&+ \Gamma_{\overline{\Psi}\chi_3\Psi}(l;k,p+q)S(p+q)\Gamma_{\overline{\Psi}\chi_3\Psi}(p+q;q,p) \\
&- \frac{2bm_\Phi^2/M_Z}{m_\Phi^2-(k+q)^2}\Gamma_{\overline{\Psi}\Phi\Psi}(l;k+q,p) \\
&+ \frac{ia}{M_Z}\left(\Gamma_{\overline{\Psi}\chi_3\Psi}(l-q;k,p)-\Gamma_{\overline{\Psi}\chi_3\Psi}(l;k,p+q)\right)
\end{aligned}
\qquad (4-39)
$$

The final additive term in (4-39), corresponding to a 1PI contribution, is shown to vanish in section 4-5.

4-4) Weak charged current Ward identities:

The Ward identity relating the three point $\overline{\Psi}W\Psi$ vertex to the fermion propagator is derived from BRST invariance of the three point function involving the W^{+}'s Fadeev-Popov ghost field:

$$0 = \delta^{BRST} < \Psi_i(x)\bar{c}^+(y)\bar{\Psi}_l(z) >$$

$$= -i\lambda\frac{g}{2\sqrt{2}}(1-\gamma_5) < \Psi_l(x)c^-(x)\bar{c}^+(y)\bar{\Psi}_l(z) >$$

$$+\lambda < \Psi_i(x)\left[\frac{1}{\xi_W}\partial^\mu W_\mu^+(y) + M_W\chi^+(y)\right]\bar{\Psi}_l(z) >$$

$$+ i\lambda\frac{g}{2\sqrt{2}} < \Psi_i(x)c^-(z)\bar{c}^+(y)\bar{\Psi}_i(z) > (1+\gamma_5). \qquad (4-40)$$

Application of the operator $(\Delta_y + \xi_W M_W^2)$ to both sides of (4-40) yields the momentum space relationship

$$(-k^2 + \xi_W M_W^2)\frac{ik^\mu}{\xi_W} G^\mu_{\Psi_i W^- \bar{\Psi}_l}(q;k,p)$$

$$+ (-k^2 + \xi_W M_W^2) M_W G_{\Psi_i \chi^- \bar{\Psi}_l}(q;k,p)$$

$$= \frac{ig}{2\sqrt{2}}\left[(1-\gamma_5)G_{\Psi_l\bar{\Psi}_l}(q-k;p) - G_{\Psi_i\bar{\Psi}_i}(q;p+k)(1+\gamma_5)\right] \qquad (4-41)$$

In terms of the truncated 3 point vertex function

$$G^\mu_{\Psi_i W^- \bar{\Psi}_l}(q;k,p) = S_i(q)\Delta_W^{\mu\nu}(k) g_{\nu\tau}\Gamma^\tau_{\Psi_i W^- \Psi_l}(q;k,p)S_l(p), \qquad (4-42)$$

we obtain the following identity through use of the W-propagator analogue of (4-23b):

$$-ik_\mu\Gamma^\mu_{\Psi_i W^- \Psi_l}(q;k,p) + M_W\Gamma_{\Psi_i \chi^- \Psi_l}(q;k,p)$$

$$= \frac{ig}{2\sqrt{2}}[S_i^{-1}(q)(1-\gamma_5) - (1+\gamma_5)S_l^{-1}(p)], \qquad (4-43a)$$

where the fermion-field subscripts I denote respectively the bottom and top members of the fermion SU(2) doublet, and where χ^- is the scalar partner of W^- [our convention is that charge flows with momentum]. Both (4-43a) and its W^+ analogue

$$-ik_\mu \Gamma^\mu_{\Psi_l W^- \Psi_t}(q;k,p) + M_W \Gamma_{\Psi_l \chi^- \Psi_t}(q;k,p)$$

$$= \frac{ig}{2\sqrt{2}}[S_l^{-1}(q)(1-\gamma_5) - (1+\gamma_5)S_t^{-1}(p)], \qquad (4-43b)$$

which are quite easily seen to be upheld by tree level Feynman rules, provide the mechanism by which 3 and 4 point functions in the W sector acquire sensitivity to quark self-energy contributions from the QCD vacuum.

The W-sector analog to (4-27) is obtained from BRST invariance of the configuration space 4 point function in the W sector corresponding to (4-25):

$$0 = \delta^{BRST} < \Psi_l(x) W^-_\mu \bar{c}^+(w) \overline{\Psi}_l(z) >$$

$$= -i\lambda \frac{g}{2\sqrt{2}}(1-\gamma_5) < \Psi_t(x) W^-_\mu(y) c^+(x) \bar{c}^+(w) \overline{\Psi}_l(z) >$$

$$-\lambda < \Psi_l(x)(\partial_\mu c^-(y)) \bar{c}^+(w) \overline{\Psi}_l(z) >$$

$$+\lambda < \Psi_l(x) W^-_\mu \left[\frac{1}{\xi_W} \partial^\nu W^+_\nu(w) + M_W \chi^+(w) \right] \overline{\Psi}_l(z) >$$

$$+ i\lambda \frac{g}{2\sqrt{2}} < \Psi_l(x) W^-_\mu(y) c^-(z) \bar{c}^+(w) \overline{\Psi}_t(z) > (1+\gamma_5). \qquad (4-44)$$

Application of $\Delta_W + \xi_W M_W^2$ to both sides of (4-44) yields (to lowest contributing order in g) the following momentum space relationship:

$$(-q^2+\xi_W M_W^2)\left(\frac{iq_\nu}{\xi_W}\right)G^{\mu\nu}_{\Psi_l W^- W^+ \overline{\Psi}_l}(l;k,q,p)$$

$$+(-q^2+\xi_W M_W^2)M_W G^{\mu}_{\Psi_l W^- \chi^+ \overline{\Psi}_l}(l;k,q,p)$$

$$=[ik^\mu G_{\Psi_l\overline{\Psi}_l}(l;p)\delta^4(p+k)$$

$$-(-q^2+\xi_W M_W^2)\left(\frac{iq_\nu}{\xi_W}\right)G_{\Psi_l\overline{\Psi}_l}(l;p)G^{\mu\nu}_{W^+W^-}(-k;q)]$$

$$+ik^\mu G_{\Psi_l c^- \overline{c}^+ \overline{\Psi}_l}(l;k,q,p)$$

$$-\frac{ig}{2\sqrt{2}}G^{\mu}_{\Psi_l W^- \overline{\Psi}_l}(l;k,p+q)(1+\gamma_5). \qquad (4-45)$$

The square bracketed term on the right hand side of (4-45) vanishes via (4-23b) $[Z \to W]$. The resulting identity for truncated momentum space Green's functions is given by

$$-iq_\nu\,\Gamma^{\mu\nu}_{\overline{\Psi}_l W^+ W^- \Psi_l}(l;k,q,p)+M_W\Gamma^{\mu}_{\overline{\Psi}_l W^+ \chi^- \Psi_l}(l;k,q,p)$$

$$=-i\left(\frac{k^\mu}{\xi_W}\right)\Gamma_{\overline{\Psi}_l \overline{c}^+ c^- \Psi_l}(l;k,q,p)$$

$$-\frac{ig}{2\sqrt{2}}\Gamma^{\mu}_{\overline{\Psi}_l W^+ \Psi_l}(l;k,p+q)S_l(p+q)(1+\gamma_5)S_l^{-1}(p). \qquad (4-46)$$

Similarly, we obtain the identity

$$- iq_\nu \Gamma^{\mu\nu}_{\overline{\Psi}_l W^- W^+ \Psi_l}(l;k,q,p) + M_W \Gamma^{\mu}_{\overline{\Psi}_l W^- \chi^+ \Psi_l}(l;k,q,p)$$

$$= -i\left(\frac{k^\mu}{\xi_W}\right)\Gamma_{\overline{\Psi}_l \overline{c}^- c^+ \Psi_l}(l;k,q,p)$$

$$+i\frac{g}{2\sqrt{2}}S_l^{-1}(l)(1-\gamma_5)S_i(i-q)\Gamma^{\mu}_{\overline{\Psi}_i W^- \Psi_l}(l-q;k,p) \qquad (4-47)$$

from BRST invariance of $<\Psi_l(x)W^+_\mu(y)\bar{c}^-(w)\overline{\Psi}_l(z)>$. For W sector 4 point functions, the W sector analog of (4-33) is obtained by replacing the $\Phi\bar{c}^Z c^Z$ vertex with the $\Phi\bar{c}^+ c^-$ (or $\Phi\bar{c}^- c^+$) vertex, leading to the relationship

$$\Gamma_{\overline{\Psi}_l \bar{c}^+ c^- \Psi_l}(l;k,q,p) = \Gamma_{\overline{\Psi}_l \bar{c}^- c^+ \Psi_l}(l;k,q,p)$$

$$= -\frac{(\xi_W g M_W/2)}{m_\Phi^2 - (k+q)^2}\Gamma_{\overline{\Psi}_l \Phi \Psi_l}(l;k+q,p) \qquad (4-48)$$

represented graphically in fig. 12.

Therefore, $\overline{\Psi}\bar{c}c\Psi$ vertices in (4-46) and (4-47) can be related to the $\overline{\Psi}\Phi\Psi$ Green's function via (4-48). One can also use (4-43a,b) to rewrite the inverse fermion propagator on the right hand sides of (4-46) and (4-47) as follows:

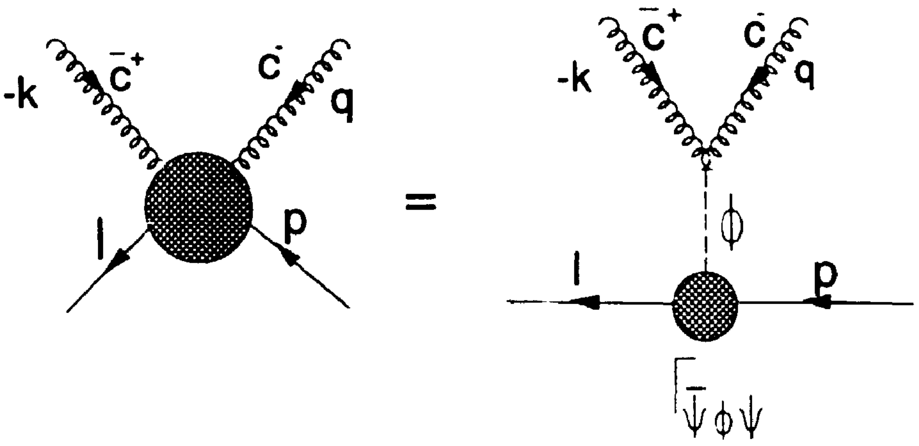


Figure 12: The $\overline{\Psi}_l \bar{c}^+ c^- \Psi_l$ truncated Green's function.

$$-\frac{ig}{2\sqrt{2}}(1+\gamma_5)S_l^{-1}(p) = -iq_\nu \Gamma^\nu_{\overline{\Psi}_l W^- \Psi_l}(p+q;q,p)$$

$$+M_W \Gamma_{\overline{\Psi}_l \chi^- \Psi_l}(p+q;q,p) - \frac{ig}{2\sqrt{2}} S_l^{-1}(p+q)(1-\gamma_5), \quad (4-49)$$

$$\frac{ig}{2\sqrt{2}} S_l^{-1}(l)(1-\gamma_5) = -iq_\mu \Gamma^\mu_{\overline{\Psi}_l W^- \Psi_l}(l;q,l-q)$$

$$+M_W \Gamma_{\overline{\Psi}_l \chi^- \Psi_l}(l;q,l-q) + i\frac{g}{2\sqrt{2}}(1+\gamma_5)S_l^{-1}(l-q). \quad (4-50)$$

If we make these substitutions into (4-46) and (4-47) and add the two equations, we obtain the relationship [l-q=p+k]

$$-iq_\nu\left[\Gamma^{\mu\nu}_{\bar\Psi_l W^- W^+ \Psi_l}(l;k,q,p)+\Gamma^{\mu\nu}_{\bar\Psi_l W^+ W^- \Psi_l}(l;k,q,p)\right]$$

$$+M_W\{\Gamma^{\mu}_{\bar\Psi_l W^- \chi^+ \Psi_l}(l;k,q,p)+\Gamma^{\mu}_{\bar\Psi_l W^+ \chi^- \Psi_l}(l;k,q,p)\}$$

$$=-iq_\nu[\Gamma^{\nu}_{\bar\Psi W^+ \Psi_l}(l;q,p+k)S_l(p+k)\Gamma^{\mu}_{\bar\Psi_l W^- \Psi_l}(p+k;k,p)$$

$$+\Gamma^{\mu}_{\bar\Psi_l W^+ \Psi_l}(l;k,p+q)S_l(p+q)\Gamma^{\nu}_{\bar\Psi_l W^- \Psi_l}(p+q;q,p)$$

$$+2\left(\frac{gM_W}{m_\Phi^2-(k+q)^2}\right)g^{\mu\nu}\Gamma_{\bar\Psi_l \Phi \Psi_l}(l;k+q,p)]$$

$$+M_W\{\Gamma_{\bar\Psi_l \chi^+ \Psi_l}(l;q,p+k)S_l(p+k)\Gamma^{\mu}_{\bar\Psi_l W^- \Psi_l}(p+k;k,p)$$

$$+\Gamma^{\mu}_{\bar\Psi_l W^+ \Psi_l}(l;k,p+q)S_l(p+q)\Gamma_{\bar\Psi_l \chi^- \Psi_l}(p+q;q,p)$$

$$+2\frac{(ig/2)}{m_\Phi^2-(k+q)^2}(2q+k)^\mu\Gamma_{\bar\Psi_l \Phi \Psi_l}(l;k+q,p)\}$$

$$+\frac{ig}{2\sqrt{2}}(1+\gamma_5)\Gamma^{\mu}_{\bar\Psi_l W^- \Psi_l}(p+k;k,p)$$

$$-i\frac{g}{2\sqrt{2}}\Gamma^{\mu}_{\bar\Psi_l W^+ \Psi_l}(l;k,p+q)(1-\gamma_5) \qquad (4-51)$$

Equation (4-51) is the W-sector analogue of (4-34). The 1PR contributions to the $\bar\Psi WW\Psi$ 4-point function are in square brackets and the 1PR contributions to the $\bar\Psi W\chi\Psi$ 4 point function are in curly brackets. The remaining unbracketed term on the right hand side of (4-51) is a 1PI contribution which, by analogy to (4-35) and (4-18), we assign to $\bar\Psi WW\Psi$. These results are delineated in the following identities:

$$q_\nu\left[\Gamma^{\mu\nu}_{\bar{\Psi}_l W^- W^+ \Psi_l}(l;k,q,p)+\Gamma^{\mu\nu}_{\bar{\Psi}_l W^+ W^- \Psi_l}(l;k,q,p)\right]$$

$$=q_\nu[\Gamma^{\nu}_{\bar{\Psi}_l W^- \Psi_l}(l;q,p+k)S_l(p+k)\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(p+k;k,p)$$

$$+\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(l;k,p+q)S_l(p+q)\Gamma^{\nu}_{\bar{\Psi}_l W^- \Psi_l}(p+q,q,p)$$

$$+\frac{2gM_W g^{\mu\nu}}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}_l \Phi \Psi_l}(l;k+q,p)]$$

$$+\frac{g}{2\sqrt{2}}\left[\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(l;k,p+q)(1-\gamma_5)-(1+\gamma_5)\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(p+k;k,p)\right]$$

$$(4-52)$$

$$\Gamma^{\mu}_{\bar{\Psi}_l W^- \chi^- \Psi_l}(l;k,q,p)+\Gamma^{\mu}_{\bar{\Psi}_l W^+ \chi^- \Psi_l}(l;k,q,p)$$

$$=\Gamma_{\bar{\Psi}_l \chi^- \Psi_l}(l;q,p+k)S_l(p+k)\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(p+k;k,p)$$

$$+\Gamma^{\mu}_{\bar{\Psi}_l W^- \Psi_l}(l;k,p+q)S_l(p+q)\Gamma^{\mu}_{\bar{\Psi}_l \chi^- \Psi_l}(p+q;q,p)$$

$$+\frac{ig(k+2q)^\mu}{m_\Phi^2-(k+q)^2}\Gamma_{\bar{\Psi}_l \Phi \Psi_l}(l;k+q,p) \qquad (4-53)$$

These results are represented graphically in figs. 13 and 14.

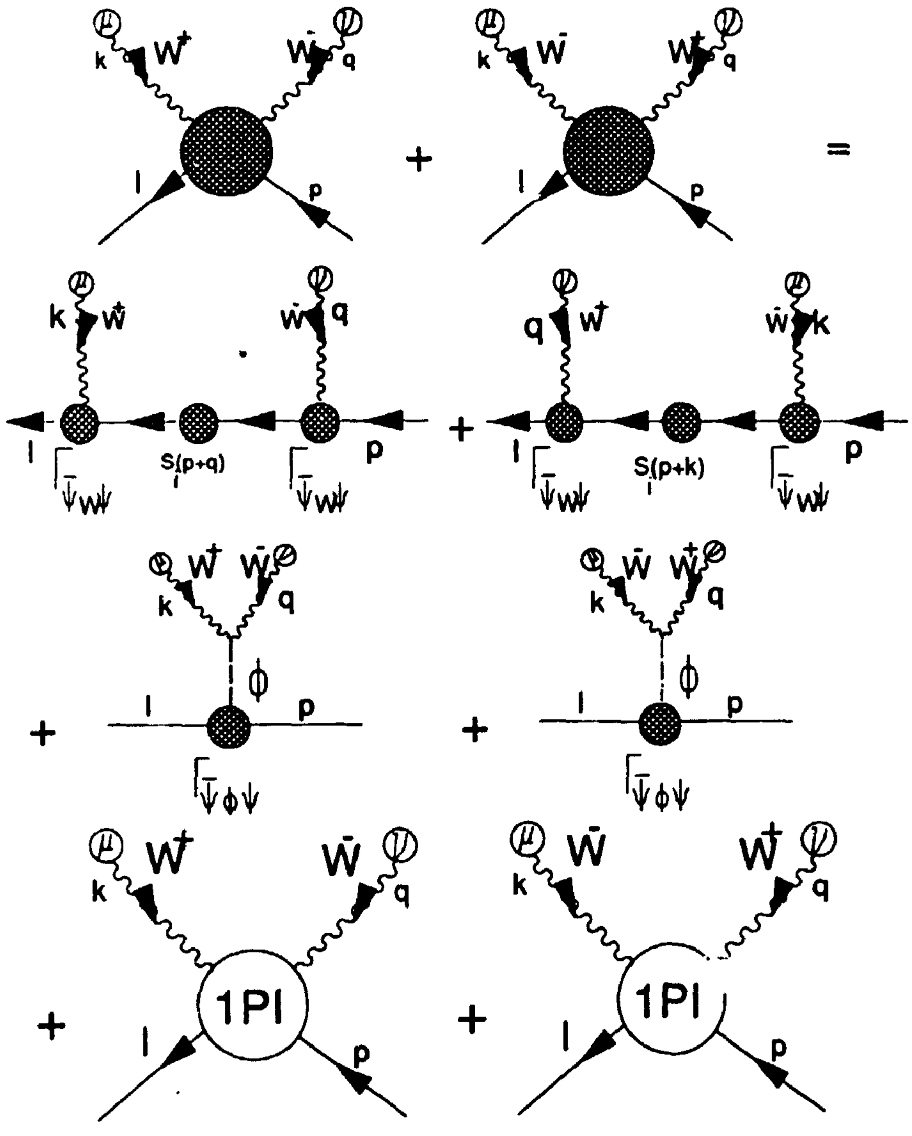


Figure 13: 1PR and 1PI components of the $\overline{\Psi}_l WW \Psi_l$ truncated Green's function.

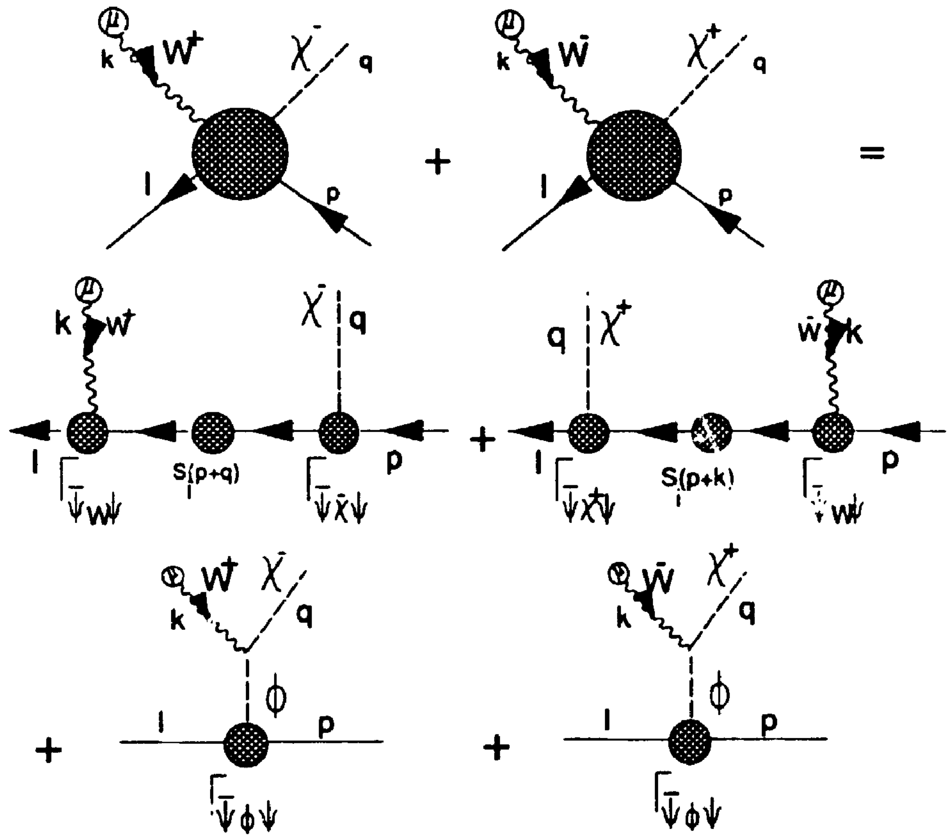


Figure 14: 1PR components of the $\overline{\Psi}_l W^{\pm}\chi^{\mp}\Psi_l$ truncated Green's function.

Finally, the W-sector analogue of (4-39) is obtained from the BRST invariance of the following configuration space 4-point function:

$$0 = \delta^{BRST} < \Psi_l(x)\chi^-(y)\bar{c}^+(w)\overline{\Psi}_l(z) >$$

$$= -i\lambda \frac{g}{2\sqrt{2}}(1-\gamma_5) < \Psi_i(x)\chi^-(y)c^+(x)\bar{c}^+(w)\overline{\Psi}_l(z) >$$

$$-\lambda \frac{g}{2} < \Psi_l(x)[<\Phi> + \Phi(y) + \chi_3(y)]c^-(y)\bar{c}^+(w)\overline{\Psi}_l(z) >$$

$$+\lambda < \Psi_l(x)\chi^-(y)\left[\frac{1}{\xi_W}\partial^\nu W_\nu^+(w) + M_W\chi^+(w)\right]\overline{\Psi}_l(z) >$$

$$+i\lambda \frac{g}{2\sqrt{2}} < \Psi_l(x)\chi^-(y)c^-(z)\bar{c}^+(w)\overline{\Psi}_l(z) > (1+\gamma_5). \quad (4-54)$$

The first term on the right hand side of (4-54) is a connected 5 point function, which is necessarily higher order in electroweak coupling than the lowest order contributions of the remaining terms. Keeping terms only to the lowest contributing electroweak order, we find that

$$\frac{1}{\xi_W}\partial_W^\nu(\Delta_W+\xi_W M_W^2)<\Psi_l(x)\chi^-(y)W_\nu^+(w)\overline{\Psi}_l(z)>$$

$$+M_W(\Delta_W+\xi_W M_W^2)<\Psi_l(x)\chi^-(y)\chi^+(w)\overline{\Psi}_l(z)>_c$$

$$+[M_W(\Delta_W+\xi_W M_W^2)<\Psi_l(x)\overline{\Psi}_{(z)}><\chi^-(y)\chi^+(w)>$$

$$-\frac{g}{2}<\Phi><\Psi_l(x)\overline{\Psi}_l(z)>\delta^4(y-w)]$$

$$=(\Delta_W+\xi_W M_W^2)\frac{g}{2}<\Phi><\Psi_l(x)c^-(y)\overline{c}^+(w)\overline{\Psi}_l(z)>_c$$

$$+\frac{g}{2}<\Psi_l(x)\Phi(y)\overline{\Psi}_l(z)>\delta^4(y-w)$$

$$+\frac{g}{2}<\Psi_l(x)\chi_3(y)\overline{\Psi}_l(z)>\delta^4(y-w)$$

$$-\frac{ig}{2\sqrt{2}}<\Psi_l(x)\chi^-(y)\overline{\Psi}_i(z)>\delta^4(z-w)(1+\gamma_5). \qquad (4-55)$$

The two terms within the square brackets on the left hand side of (4-55) cancel $\left[\frac{g<\Phi>}{2}=M_W\right]$. If we evaluate (4-55) in terms of truncated momentum space Green's functions, we find [making appropriate use of (4-23) analogs for W-sector propagators] that

$$-iq_\nu\Gamma^\nu_{\Psi_l\chi^+W^-\Psi_l}(l;k,q,p)+M_W\Gamma_{\Psi_l\chi^+\chi^-\Psi_l}(l;k,q,p)$$

$$=M_W\Gamma_{\Psi_l\bar{c}^+c^-\Psi_l}(l;k,q,p)$$

$$+\frac{g}{2}\frac{\xi_W M_W^2-k^2}{m_\Phi^2-(k+q)^2}\Gamma_{\Psi_l\Phi\Psi_l}(l;k+q,p)$$

$$+\frac{g}{2}\frac{\xi_W M_W^2-k^2}{\xi_Z M_Z^2-(k+q)^2}\Gamma_{\Psi_l\chi_3\Psi_l}(l;k+q,p)$$

$$-\frac{ig}{2\sqrt{2}}\Gamma_{\Psi_l\chi^-\Psi_l}(l;k,p+q)S_i(p+q)(1+\gamma_5)S_l^{-1}(p). \qquad (4-56)$$

One can obtain an analog to (4-56) through BRST invariance of $<\Psi_l(x)\chi^+(y)\bar{c}^-(w)\overline{\Psi}_l(z)>$, similar to (4-54). The sum of (4-56) with its "charge conjugate" yields the relationship

$$-iq_\nu\left[\Gamma^\nu_{\Psi_l\chi^+W^-\Psi_l}(l;k,q,p)+\Gamma^\nu_{\Psi_l\chi^-W^+\Psi_l}(l;k,q,p)\right]$$

$$+M_W\left[\Gamma_{\Psi_l\chi^+\chi^-\Psi_l}(l;k,q,p)+\Gamma_{\Psi_l\chi^-\chi^+\Psi_l}(l;k,q,p)\right]$$

$$=M_W\left[\Gamma_{\Psi_l\bar{c}^+c^-\Psi_l}(l;k,q,p)+\Gamma_{\Psi_l\bar{c}^-c^+\Psi_l}(l;k,q,p)\right]$$

$$+g\frac{\xi_W M_W^2-k^2}{m_\Phi^2-(k+q)^2}\Gamma_{\Psi_l\Phi\Psi_l}(l;k+q,p)$$

$$+\frac{ig}{2\sqrt{2}}S_l^{-1}(l)(1-\gamma_5)S_i(p+k)\Gamma_{\Psi_l\chi^-\Psi_l}(p+k;k,p)$$

$$-\frac{ig}{2\sqrt{2}}\Gamma_{\Psi_l\chi^-\Psi_l}(l;k,p+q)S_i(p+q)(1+\gamma_5)S_l^{-1}(p). \qquad (4-57)$$

We now substitute (4-48), (4-49) and (4-50) into the right hand side of (4-57), and (4-53) into the left hand side of (4-57) to obtain (fig. 15):

$$
\begin{aligned}
&\Gamma_{\bar{\Psi}_l \chi^+ \chi^- \Psi_l}(l;k,q,p) + \Gamma_{\bar{\Psi}_l \chi^- \chi^+ \Psi_l}(l;k,q,p) \\
&= \Gamma_{\bar{\Psi}_l \chi^+ \Psi_i}(l;q,p+k) S_i(p+k) \Gamma_{\bar{\Psi}_i \chi^- \Psi_l}(p+k;k,p) \\
&+ \Gamma_{\bar{\Psi}_l \chi^- \Psi_i}(l;k,p+q) S_i(p+q) \Gamma_{\bar{\Psi}_i \chi^- \Psi_l}(p+q,q,p) \\
&- 2\frac{g m_\Phi^2 / 2M_W}{m_\Phi^2 - (k+q)^2} \Gamma_{\bar{\Psi}_l \Phi \Psi_l}(l;k+q,p).
\end{aligned}
\tag{4-58}
$$

4-5) 1PI contributions and Landau gauge:

In the presence of an external self-energy contribution to S_i and S_l, we can retain a distinction between the pole of the propagator (4-1) and the lagrangian quark mass resulting from Yukawa interactions with the vacuum expectation value $< \Phi >$. This latter mass characterizes the lowest order contribution to the 3-point function

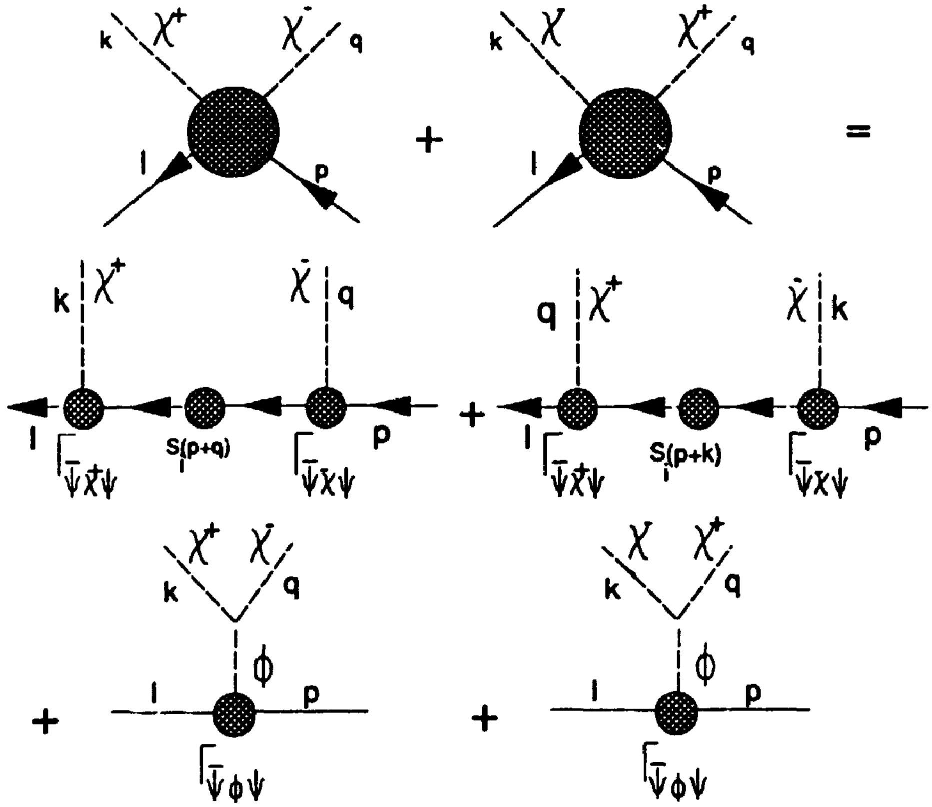


Figure 15: 1PR components of the $\overline{\Psi}_l\chi^{\pm}\chi^{\mp}\Psi_l$ truncated Green's function.

$$\left[\Gamma_{\overline{\Psi}_l\phi\Psi_l}(l;k+q,p)\right]_{tree} = -\frac{2m_l^c b}{M_Z}. \qquad (4-59)$$

None of the identities we have derived relating 3 point functions to 2 point functions precludes the maintenance of (4-59) in the presence of self-energy mass contributions external to $SU(2)\times U(1)$. In other words, we are free to continue identifying the lagrangian quark mass with the product of the $|\Gamma_{\overline{\Psi}\phi\Psi}|$ Yukawa coupling and the vacuum expectation value

$<\Phi>$; (4-59) is uncorrected by externally generated self-energies to lowest contributing order in the electroweak coupling. If (4-38) is to be upheld, however, we see that $\Gamma_{\overline{\Psi}_\chi,\Psi}$ must also be unaffected by externally generated mass contributions to the self-energy. Moreover, the absence of 1PI contributions to (4-58), (4-48) and (4-53) require the following analog to (4-38):

$$\Gamma_{\overline{\Psi}_l \Phi \Psi_l}(l;k+q,p) = -\frac{i}{2\sqrt{2}}\Big[(1+\gamma_5)\Gamma_{\overline{\Psi}_l \chi^- \Psi_l}(p+k;k,p) - \Gamma_{\overline{\Psi}_l \chi^- \Psi_l}(l;k,p+q)(1-\gamma_5)\Big]. \tag{4-60}$$

If $\Gamma_{\overline{\Psi}\Phi\Psi}$ is to be unaffected by external self-energy mass contributions, (4-60) is upheld provided $\Gamma_{\overline{\Psi}_\chi^-\Psi}, \Gamma_{\overline{\Psi}_\chi^-\Psi}$ are similarly unaffected. Thus, the potential 1PI contributions to (4-39), corresponding to those terms on the right hand side with coefficient "a", are seen to vanish. [Similarly, 1PI terms are omitted in (4-58)]. Furthermore, we see from (4-24) and (4-43) that all external contributions to quark inverse propagators must be absorbed entirely by the gauge boson vertices, as opposed to the scalar partner vertices. This property is, of course, upheld for unbroken symmetry gauge boson vertices as well, as evidenced by the relation between the QED vertex of (4-9) and the Fermion inverse propagators appearing in that same expression.

Thus external self-energy contributions, particularly those originating from the coupling of electroweak interactions to an $SU(2)\times U(1)$ noninvariant QCD vacuum, are seen to modify only the $\overline{\Psi}A\Psi, \overline{\Psi}Z\Psi, \overline{\Psi}W^{\pm}\Psi$ subset of electroweak 3 point functions. These modifications are then seen to generate additional self-energy sensitive contributions to the $\overline{\Psi}AA\Psi, \overline{\Psi}ZZ\Psi, \overline{\Psi}W^{\mp}W^{\pm}\Psi$ subset of electroweak 4 point functions that would not be expected from the purely 1PR contributions to those functions anticipated from the Feynman rules. These additional 1PI contributions are delineated in (4-18), (4-35) and (4-52). On the other hand, these corrected 3- and 4-point Green's functions may be put in an explicit form which satisfies their relevant Ward identities. For example, $\overline{\Psi}Z\Psi$ and $\overline{\Psi}ZZ\Psi$ can be written as

$$\Gamma^{\mu}_{\overline{\Psi}Z\Psi}(p+k;k,p) = \gamma^{\mu}(a-b\gamma_5) + \frac{k^{\mu}}{k^2}[\sigma((p+k)^2)(a-b\gamma_5) - (a+b\gamma_5)\sigma(p^2)]; \quad (4-61)$$

$$\Gamma^{\mu\nu}_{\Psi ZZ\Psi}(p+k+q;k,q,p)$$

$$=\gamma^{\nu}(a-b\gamma_5)S(p+k)\gamma^{\mu}(a-b\gamma_5)$$

$$+\gamma^{\mu}(a-b\gamma_5)S(p+q)\gamma^{\nu}(a-b\gamma_5)$$

$$-\left(\frac{2mb}{M_Z}\right)\frac{1}{m_{\phi}^2-(k+q)^2}(4bM_Z g^{\mu\nu})$$

$$+\frac{q^{\nu}}{q^2}\{[\sigma((p+k+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+k)^2)]S(p+k)\gamma^{\mu}(a-b\gamma_5)$$

$$+\gamma^{\mu}(a-b\gamma_5)S(p+q)[\sigma((p+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]\}$$

$$+\frac{k^{\mu}}{k^2}\{\gamma^{\nu}(a-b\gamma_5)S(p+k)[\sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]\}$$

$$+[\sigma((p+k+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+q)^2)]S(p+q)\gamma^{\nu}(a-b\gamma_5)\}$$

$$+\frac{q^{\nu}k^{\mu}}{q^2k^2}\{[\sigma((p+k+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+k)^2)]S(p+k)$$

$$\times[\sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+[\sigma((p+k+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+q)^2)]S(p+q)$$

$$\times[\sigma((p+q)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+(a-b\gamma_5)^2\sigma((p+k+q)^2)+(a+b\gamma_5)^2\sigma(p^2)$$

$$-(a^2-b^2)[\sigma((p+q)^2)+\sigma((p+k)^2)]\}. \qquad (4-62)$$

Although other kinematic structures may also be consistent with (4-24) and (4-35), those chosen for (4-61) and (4-62) are seen to have all vertex σ-dependence in terms that are annihilated by transverse projection operators. In other words, all σ-dependent departure from tree level vertices (including the 1PI contribution to fig. 9) are seen *to vanish in Landau gauge calculations*. This result implies that a Landau gauge calculation with naive electroweak Feynman rules will yield the "correct" result, even when quark propagators S(p) contain externally generated self-energies of nonperturbative origin, as in (4-1)[36].

In the next chapter, we test the modifications to electroweak Green's functions we have obtained in arbitrary covariant gauge by examining the electroweak gauge-parameter dependence of on-shell self-energy amplitudes in the presence of externally-generated contributions $\sigma(p^2)$ to the quark propagator.

CHAPTER FIVE

TEST OF SELF-ENERGY-MODIFIED ELECTROWEAK GREEN'S FUNCTIONS

5-1) Gauge independence of purely perturbative 2-point functions:

As we have already argued in chapter 3, gauge parameter independence of physical (i.e. on mass shell) Green's functions is essential for the gauge invariance of physically measurable processes. This on shell gauge parameter independence also characterizes on-mass-shell two point functions.

For example, the ξ_W gauge parameter of *purely perturbative* $SU(2)\times U(1)$ theory enters the d quark two point function through the ξ_W –sensitive contributions of fig. 16. The gauge parameter dependence of fig. 16 first two contributions (labelled a and b) can be ascertained by utilizing only the gauge parameter sensitive piece of the W propagator $[\{k^\mu k^\nu/[M_W^2(k^2-\xi_W M_W^2)]\}]$, the respective contribution of these graphs are then given (neglecting third generation mixings) by (the superscript c of the lagrangian mass m^c has been omitted for convenience)

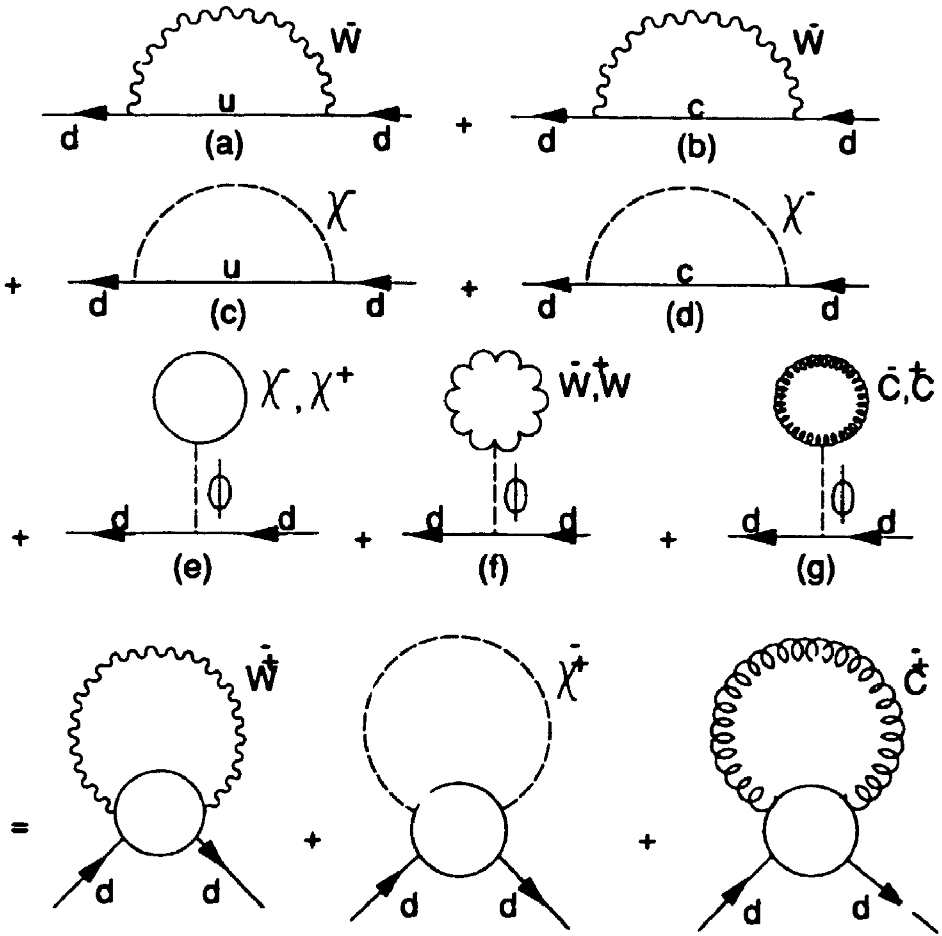


Figure 16: Purely perturbative electroweak contributions to the d quark two point function in the absence of external nonperturbative contributions to the quark inverse propagator.

$$\sigma_a^{\xi_W}(p) = \bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\left(\frac{g\cos\theta_c}{2\sqrt{2}}\right)\gamma^\mu(1-\gamma_5)(m_u-\hat{p}+\hat{k})^{-1}$$

$$\gamma^\nu(1-\gamma_5)\left(\frac{g\cos\theta_c}{2\sqrt{2}}\right)\frac{k_\mu k_\nu}{M_W^2(k^2-\xi_W M_W^2)}d(p)$$

$$=\frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\frac{(1+\gamma_5)\hat{k}(m_u-\hat{p}+\hat{k})^{-1}\hat{k}(1-\gamma_5)}{k^2-\xi_W M_W^2}d(p),$$

$$(5-1)$$

or

$$\sigma_a^{\xi_W}(p) = \frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}[(m_u-\hat{p}+\hat{k}-(m_d-\hat{p})+m_d-m_u)$$

$$+\gamma_5(m_u-\hat{p}+\hat{k}+(m_d+\hat{p})-(m_u+m_d))]\,(m_u-\hat{p}+\hat{k})^{-1}\,[(m_u-\hat{p}+\hat{k}-(m_d-\hat{p})+m_d-m_u)$$

$$-(m_u-\hat{p}+\hat{k}+(m_d+\hat{p})-(m_u+m_d))\gamma_5)]\frac{1}{k^2-\xi_W M_W^2}d(p). \qquad (5-2)$$

Using the on shell condition

$$\bar{d}(p)S^{-1}(p)=\bar{d}(p)(m_d-\hat{p})=S^{-1}(p)d(p)=(m_d-\hat{p})d(p)=0 \qquad (5-3)$$

we obtain

$$\sigma_a^{\xi_w}(p) = -\frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\{(m_u - m_d) + (m_d + m_u)\gamma_5\}$$

$$(m_u - \hat{p} + \hat{k})^{-1}\{(m_d - m_u) + (m_d + m_u)\gamma_5)\}\left[\frac{1}{k^2 - \xi_W M_W^2}\right]d(p)$$

$$+\frac{g^2\cos^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\frac{2(1+\gamma_5)\hat{k}}{k^2 - \xi_W M_W^2}d(p)$$

$$+\{\frac{g^2 m_d\cos^2\theta_c}{4M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\frac{(1-\gamma_5)}{k^2 - \xi_W M_W^2}d(p)\}. \qquad (5-4)$$

The second term in (5-4) vanishes due to symmetric integration. The ξ_W –sensitive part of fig. 16(b) can be evaluated in a similar manner:

$$\sigma_b^{\xi_w}(p) = -\frac{g^2\sin^2\theta_c}{8M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\{(m_c - m_d) + (m_d + m_c)\gamma_5\}$$

$$(m_c - \hat{p} + \hat{k})^{-1}\{(m_d - m_c) + (m_d + m_c)\gamma_5)\}\left[\frac{1}{k^2 - \xi_W M_W^2}\right]d(p)$$

$$+\{\frac{g^2 m_d\sin^2\theta_c}{4M_W^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}\frac{(1-\gamma_5)}{k^2 - \xi_W M_W^2}d(p)\}. \qquad (5-5)$$

The W's scalar partner $\chi^{\pm}$ also has a gauge dependent propagator $[1/(\xi_W M_W^2 - k^2)]$; the c, d, and e contributions of fig. 16 are given by

$$\sigma_c^{\xi_W}(p) = \bar{d}(p)\int \frac{d^4k}{i(2\pi)^4}\left(-i\frac{g\cos\theta_c}{2\sqrt{2}M_W}\{(m_u - m_d) + (m_u + m_d)\gamma_5\}\right)$$

$$(m_u - \hat{p} + \hat{k})^{-1}\left(-i\frac{g\cos\theta_c}{2\sqrt{2}M_W}\{(m_d - m_u) + (m_u + m_d)\gamma_5\}\right)\left[\frac{1}{\xi_W M_W^2 - k^2}\right]d(p),$$

$$(5-6)$$

$$\sigma_d^{\xi_W}(p) = \bar{d}(p)\int \frac{d^4k}{i(2\pi)^4}\left(-i\frac{g\sin\theta_c}{2\sqrt{2}M_W}\{(m_c - m_d) + (m_c + m_d)\gamma_5\}\right)$$

$$(m_c - \hat{p} + \hat{k})^{-1}\left(-i\frac{g\sin\theta_c}{2\sqrt{2}M_W}\{(m_d - m_c) + (m_c + m_d)\gamma_5\}\right)\left[\frac{1}{\xi_W M_W^2 - k^2}\right]d(p),$$

$$(5-7)$$

$$\sigma_e^{\xi_W}(p) = \frac{g^2 m_d}{4M_W^2}\bar{d}(p)\int \frac{d^4k}{i(2\pi)^4}\left[\frac{1}{\xi_W M_W^2 - k^2}\right]d(p), \qquad (5-8)$$

The integrands of (5-6) and (5-7) respectively cancel the first integrands on the right hand sides of (5-4) and (5-5). Moreover, the sum of the remaining (curly bracketed) contributions to (5-4) and (5-5),

$$\frac{g^2 m_d(\cos^2\theta_c + \sin^2\theta_c)}{4M_W^2}\bar{d}(p)\int \frac{d^4k}{i(2\pi)^4}\frac{(1-\gamma_5)}{k^2 - \xi_W M_W^2}d(p)$$

has a scalar component equal and opposite to $\sigma_e^{\xi_W}$ (5-8). The pseudoscalar component of this sum vanishes on the $\hat{p} = m_d$ mass shell, since for on shell d quark spinors:

$$\bar{d}(p)\gamma_5 d(p) = \frac{\bar{d}(p)\{\hat{p},\gamma_5\}d(p)}{2m_d} = 0 \qquad (5-9)$$

Consequently, the sum of the fig. 16 contributions a-e is independent of ξ_W. The f-contribution and g-contribution (involving the W's Fadeev-Popov ghost) cancel the $k^2 - \xi_W M_W^2$ denominator in the integrand, yielding an ξ_W −independent sum:

$$\sigma_f^{\xi_W}(p) + \sigma_g^{\xi_W}(p) = -\frac{g^2 m_d}{2M_W^2 m_\phi^2}\bar{d}(p)\int\frac{d^4k}{i(2\pi)^4}d(p). \qquad (5-10)$$

Thus, the ξ_W −dependence of the purely perturbative d quark two point function vanishes on the d quark mass shell:

$$\lim_{\hat{p}\to m_d}\frac{\partial}{\partial\xi_W}\left(\Sigma_{i=a}^{g}\sigma_i^{\xi_W}(p)\right) = 0. \qquad (5-11)$$

5-2) Gauge independence of electroweak 2-point functions for $\sigma(p^2) \neq 0$:

In the final line of fig. 16, the ξ_W −dependent contributions to the d quark two point function are shown to correspond to the electroweak four point functions $<\bar{d}W^+W^-d>, <\bar{d}\chi^+\chi^-d>, <\bar{d}\bar{c}^+c^-d>$ with external W, χ and C lines respectively contracted into ξ_W −dependent propagators. For the purely perturbative case, these functions (fig. 16) are all one particle reducible (1PR), as is evident from severing the ξ_W −sensitive propagator lines in drawings a-g. In the presence of additional external (nonperturbative) contributions to quark self-energies, as denoted by $\sigma(p^2)$ in (4-1), the same four point functions occurring in fig. 16 acquire σ –sensitive contributions, as discussed in the previous ch pter. Indeed, the three gauge parameters of electroweak theory enter quark self-energies through the ξ_A –sensitive contributions of fig. 17, the ξ_Z –sensitive contributions of fig. 18, and the ξ_W –sensitive contributions of fig. 19, where the (now shaded) four point functions appearing in all three figures correspond respectively to joining the non-fermionic lines of fig. 7 [leading to fig. 17], figs. 8, 9 and 11 [leading to fig. 18], and figs. 12, 13 and 15 [fig. 19].

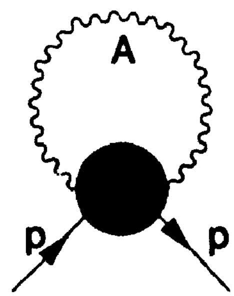


Figure 17: ξ_A –sensitive contribution to the quark two point function in the presence of external non-perturbative contributions to the quark inverse propagator.

For example, the ξ_A –sensitive contribution to fig. 17 [generated through the $(1-\xi_A)k_\tau k_\sigma/(k^2)^2$ portion of the photon propagator] is proportional to

$$\sigma_{17}^{\xi_A}(p) = (1-\xi_A)\bar{u}(p)\int \frac{d^4k}{i(2\pi)^4}\frac{k_\tau k_\sigma}{(k^2)^2}\Gamma^{\tau\sigma}_{\Psi AA\Psi}(p;k,-k,p)u(p) \qquad (5-12)$$

Upon substitution of (4-16) into (5-12) we find that

$$\sigma_{17}^{\xi_A}(p) = eQ(1-\xi_A)\int \frac{d^4k}{i(2\pi)^4 (k^2)^2} \bar{u}(p)[S^{-1}(p)S(p+k)k_\tau \Gamma^\tau_{\bar{\Psi}A\Psi}(p+k;k,p)$$

$$-k_\tau \Gamma^\tau_{\bar{\Psi}A\Psi}(p;k,p-k)S(p-k)S^{-1}(p)]u(p). \tag{5-13}$$

On the quark mass shell, $S^{-1}(p)u(p)$ and $\bar{u}(p)S^{-1}(p)$ are defined to be zero, provided the quark mass is identified with the pole of the quark propagator (4-1):

$$m_{pole} = m^c - \sigma(m^2_{pole}) \tag{5-14}$$

Consequently, $\sigma_{17}^{\xi_A}(p) = 0$; the QED four point function we obtain in fig. 7 ensures the retention of an ξ_A –independent quark two point function.

We emphasize that this gauge parameter independence relies upon the same QED Ward identity that yields the 1PI contribution discussed immediately after eq. (4-18). A potential source of confusion about the need for such 1PI contributions is the on-shell ξ_A –independence of the contributions of fig. 20, corresponding to purely 1PR contributions to the $<\bar{\Psi}AA\Psi>$ four point function.

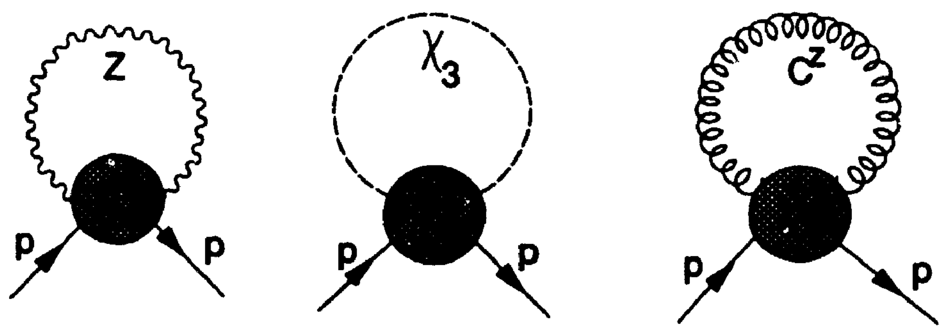


Figure 18: ξ_Z-sensitive contributions to the quark two point function in the presence of external nonperturbative contributions to the quark inverse propagator.

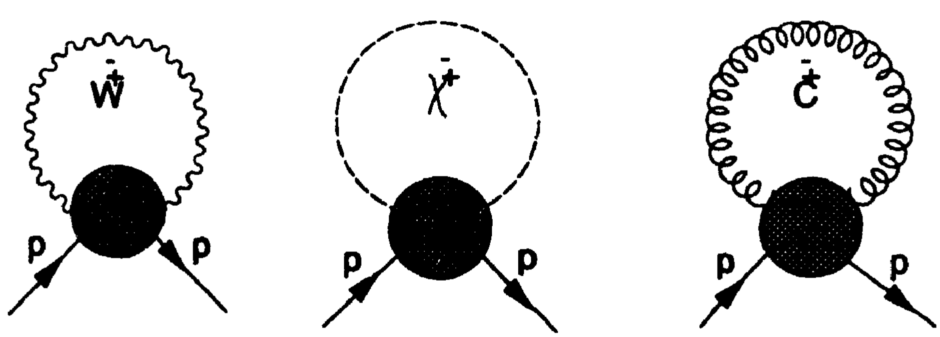


Figure 19: ξ_W-sensitive contributions to the quark two point function in the presence of external nonperturbative contributions to the quark inverse propagator.

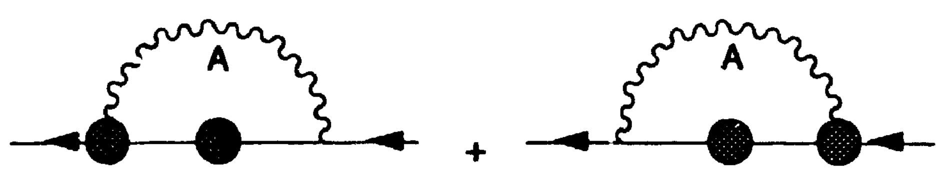


Figure 20: Usual incorporation of perturbative vertex and propagator corrections to the fermion two point function.

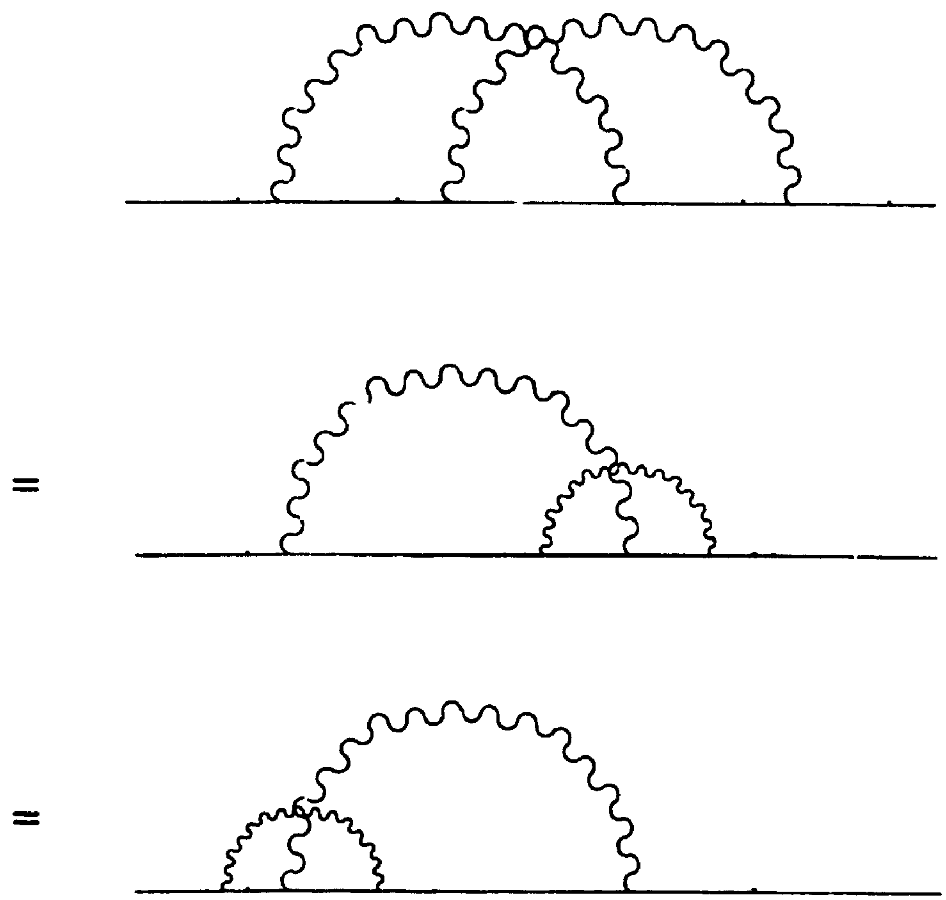


Figure 21: Demonstration of how perturbative corrections to the fermion two point function are consistent with retention of an uncorrected vertex, as in fig. 21.

The ξ_Λ –independence of fig. 20 considered on shell $[S^{-1}(p)u(p) = 0]$ is easily verified through use of the three point function Ward identity (4-9). Indeed, fig. 20 is the appropriate realization of fig. 17 for purely perturbative QED contributions to fermion two point functions, since the retention of a bare vertex is necessary to avoid double counting the (overlapping) divergences of purely perturbative amplitudes (e.g.. fig 21). However, such arguments are no longer appropriate for the corrections external to electroweak theory (such as those arising from gluon corrections). Indeed, the fig. 20 analogs (including scalar partners) for ξ_W –sensitive contributions to quark two point functions fail to exhibit gauge parameter independence in the presence of an externally generated $\sigma(p^2)$. Rather, the on shell ξ_W – and ξ_Z –independence of quark two point functions is realized *only* through careful consideration of the four point functions of figs. 18 and 19.

The appropriate Ward identities for the four point functions of fig. 8 are equations (4-33), (4-35) and (4-39), with $\Gamma_{\bar{\Psi}\Phi\Psi}$ and $\Gamma_{\bar{\Psi}\chi_3\Psi}$ constrained to their tree level values $-2m_u b/M_Z, -2im_u b\gamma_5/M_Z$, respectively, for up quarks. The gauge parameter dependent part of the fig. 18 amplitude $\left(\sigma_{18}^{\xi_Z}\right)$ is obtained through explicit use of the ξ_Z –dependent portion of the Z propagator for connecting the external Z lines in fig. 9, as well as the χ_3 and C^Z propagators for connecting the scalar partner and ghost external lines occurring in figs. 11 and 12:

$$\sigma_{18}^{\xi_Z}(p) = \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4(\xi_Z M_Z^2 - k^2) M_Z^2} \{ \frac{1}{2} [-k_\nu \Gamma^\nu_{\Psi Z \Psi}(p; -k, p+k) S(p+k) k_\mu \Gamma^\mu_{\Psi Z \Psi}(p+k; k, p)$$

$$+ k_\mu \Gamma^\mu_{\Psi Z \Psi}(p; k, p-k) S(p-k)(-k_\nu) \Gamma^\nu_{\Psi Z \Psi}(p-k; -k, p)$$

$$+ \left(-\frac{2m_u b}{M_Z}\right)\left(\frac{1}{m_\Phi^2}\right)(-4bM_Z k^2)$$

$$+ k_\mu \Gamma^\mu_{\Psi Z \Psi}(p; k, p-k)(a - b\gamma_5) - (a + b\gamma_5) k_\mu \Gamma^\mu_{\Psi Z \Psi}(p+k; k, p)]$$

$$+ \frac{1}{2}[(-2im_u b\gamma_5) S(p+k)(-2im_u b\gamma_5)$$

$$+ (-2im_u b\gamma_5) S(p-k)(-2im_u b\gamma_5)$$

$$- (2bm_\Phi^2)\left(\frac{1}{m_\Phi^2}\right)(-2m_u b)]$$

$$+ (-1)\left[\left(-\frac{2b\xi_Z M_Z^2}{m_\Phi^2}\right)(-2m_u b)\right]\} u(p)$$

$$= \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4 M_Z^2} \{ \frac{-4m_u b^2}{m_\Phi^2} \} u(p). \qquad (5-15)$$

In obtaining the last line of (5-15), we have used repeatedly the three point function Ward identity (4-24) as well as the on shell $\bar{u}(p) S^{-1}(p) = S^{-1}(p) u(p) = 0$ constraints discussed earlier. The only surviving term in (5-15) is directly analogous to (5-10) in the purely perturbative case; the integrand conspires to cancel the $(\xi_Z M_Z^2 - k^2)$ denominator to yield a net contribution that is independent of ξ_Z:

$$\frac{\partial}{\partial \xi_Z} \sigma_{19}^{\xi_Z}(p) = 0. \qquad (5-16)$$

The on shell ξ_W-independence of fig. 19 is obtained through analogous use of ξ_W-sensitive four point functions:

$$\sigma_{20}^{\xi_W}(p) = \bar{u}(p) \int \frac{d^4k}{i(2\pi)^4(\xi_W M_W^2 - k^2)M_W^2} [-k_\mu k_\nu \Big(\Gamma^{\mu\nu}_{\bar{\Psi}W^+W^-\Psi}(p;k,-k,p) + \Gamma^{\mu\nu}_{\bar{\Psi}W^-W^+\Psi}(p;k,-k,p) \Big)$$

$$+ M_W^2 (\Gamma_{\bar{\Psi}\chi^+\chi^-\Psi}(p;k,-k,p) + \Gamma_{\bar{\Psi}\chi^-\chi^+\Psi}(p;k,-k,p))$$

$$- M_W^2 (\Gamma_{\bar{\Psi}\bar{c}^+c^-\Psi}(p;k,-k,p) + \Gamma_{\bar{\Psi}\bar{c}^-c^+\Psi}(p;k,-k,p))] u(p). \qquad (5-17)$$

One finds from (4-48), (4-52) and (4-58), as well as from judicious application of the three point function Ward identities (4-43), that

$$\frac{\partial}{\partial \xi_W} \sigma_{19}^{\xi_W}(p) = 0 \qquad (5-18)$$

when the mass shell condition is imposed.

Thus, we find that the one loop corrections to quark two point functions are gauge parameter independent, both for the purely perturbative spontaneously broken $SU(2) \times U(1)$ theory and for that same theory augmented with arbitrarily momentum dependent self-energy contributions $\sigma(p^2)$ of nonperturbative origin. We stress that gauge parameter independence in the latter case would not have occurred had the 1PI contributions to (4-18), (4-35) and (4-52) been omitted; such contributions, of course, vanish if $\sigma(p^2) = 0$.

5-3) Gauge independence of the $\sigma(p^2)$-induced Yukawa 3-point function:

In the chiral limit of the electroweak lagrangian, primitive Yukawa couplings of Φ and χ_3 scalar fields necessarily vanish with the vanishing of the lagrangian quark mass. The lowest order induced Yukawa interaction in the chiral limit arises from the graphs of fig. 22, involving the four point functions of figs. 9 and 13. Similar graphs involving the four point functions of figs. 8, 10, 11, 12, 14, and 15 cannot contribute in the chiral limit, as such graphs vanish when the lagrangian Yukawa coupling is taken to be zero (i.e., $m^c = 0$).

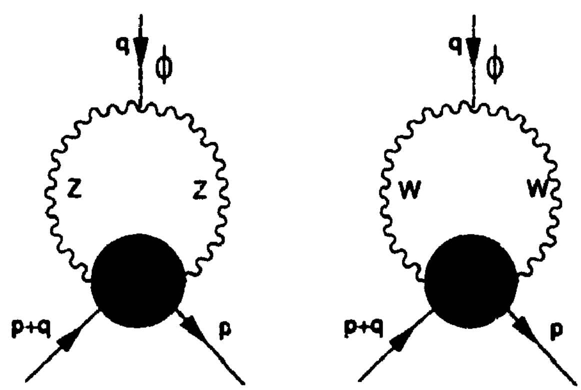


Figure 22: Induced Yukawa interaction in the chiral limit of electroweak theory.

The induced Yukawa interaction of fig. 22 is an interesting example of a three point amplitude sensitive to quark self-energy contributions external to electroweak physics. The contributions to this amplitude from internal Z lines is given by

$$\bar{u}(p+q)\Gamma^{ind}_{\Psi\Phi\Psi}(p+q;q,p)u(p)$$

$$= 2bM_Z \int \frac{d^4k}{i(2\pi)^4} \bar{u}(p+q)\Gamma^{\mu\nu}_{\Psi ZZ\Psi}(p+q;k+q,-k,p)\{\frac{g_{\nu\tau} - (1-\xi_Z)\frac{k_\nu k_\tau}{k^2-\xi_Z M_Z^2}}{k^2 - M_Z^2}\}$$

$$\{\frac{g_{\mu\tau} - (1-\xi_Z)\frac{(k+q)_\mu (k+q)_\tau}{(k+q)^2-\xi_Z M_Z^2}}{(k+q)^2 - M_Z^2}\}u(p).$$

$$(5-19)$$

The portion of this expression involving the gauge parameter ξ_Z necessarily contains factors of either $k_\nu \Gamma^{\mu\nu}_{\Psi ZZ\Psi}(p+q;k+q,-k,p)$ or $(k+q)_\mu \Gamma^{\mu\nu}_{\Psi ZZ\Psi}(p+q;k+q,-k,p)$ as coefficients of ξ_Z –sensitive quantities. These factors can be evaluated through use of (4-27), which in the $m^c = 0$ chiral limit simplifies to

$$ik_\nu \Gamma^{\mu\nu}_{\Psi ZZ\Psi}(p+q;k+q,-k,p) = iS^{-1}(p+q)(a-b\gamma_5)S(p+k+q)\Gamma^{\mu}_{\Psi Z\Psi}(p+k+q;k+q,p)$$

$$-\Gamma^{\mu}_{\Psi Z\Psi}(p+q;k+q,p-k)S(p-k)(a+b\gamma_5)S^{-1}(p) \quad (5-20)$$

We then see from (5-19) that the right hand side of (5-20) vanishes between $\bar{u}(p+q)$ and u(p) on-shell external fermion spinors, as $\bar{u}(p+q)S^{-1}(p+q)$ and $S^{-1}(p)u(p)$ both equal zero. A similar argument can be presented for the ξ_W –independence of the induced Yukawa coupling to on-shell quarks. We note that our application of the on shell condition $S^{-1}(p)u(p) = 0$ corresponds to having a dynamical quark mass m defined by the relation:

$$m = \lim_{p^2 \to m^2}(-\sigma(p^2)) \quad (5-21)$$

a relation obtained by setting the lagrangian mass m^c in (5-14) equal to zero.

5-4) Magnitude of the induced Yukawa coupling:

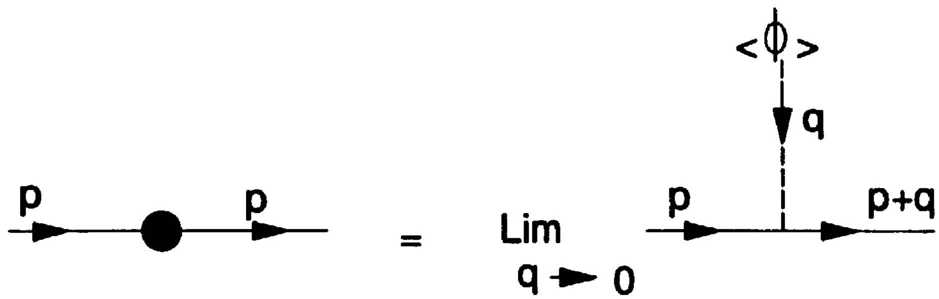


Figure 23: Relation between current mass and Yukawa coupling.

The fermion masses generated through the usual electroweak Yukawa interactions may be regarded as arising from the zero momentum transfer Yukawa coupling of the vacuum expectation value $< \Phi >$ to a massless fermion (fig. 23). In the limit of lagrangian chiral symmetry, in which lagrangian Yukawa couplings vanish, any *induced* Yukawa interaction (such as in fig. 22) necessarily will permit the occurrence of a mass via this zero momentum transfer induced coupling of a massless fermion to $< \Phi >$:

$$\bar{u}(p)\gamma^{ind}_{\Psi\Phi\Upsilon}(p;0,p)u(p) \cong -\frac{2bm^{ind}}{M_Z}$$

$$= (2bM_Z)\int\frac{d^4k}{i(2\pi)^4}\bar{u}(p)\{\Gamma^{\mu\nu}_{\Psi ZZ\Upsilon}(p;k,-k,p)\frac{g_{\mu\nu}}{(k^2-M_Z^2)^2}\}u(p)$$

$$+\frac{gM_W}{2}\int\frac{d^4k}{i(2\pi)^4}\bar{u}(p)\{\left[\Gamma^{\mu\nu}_{\Psi W^+W^-\Upsilon}(p;k,-k,p)+\Gamma^{\mu\nu}_{\Psi W^-W^+\Upsilon}(p;k,-k,p)\right]\frac{g_{\mu\nu}}{(k^2-M_W^2)^2}\}u(p). \qquad (5-22)$$

Let us consider the first integral on the right hand side of (5-22), corresponding to the first graph of fig. 22. The four point function in this integrand may be obtained directly from (4-62) taken in the chiral limit:

$$\Gamma^{\mu\mu}_{\Psi ZZ\Psi}(p;k,-k,p) = \gamma^{\mu}(a-b\gamma_5)S(p+k)\gamma_{\mu}(a-b\gamma_5) + \gamma^{\mu}(a-b\gamma_5)S(p-k)\gamma_{\mu}(a-b\gamma_5)$$

$$-\frac{k^{\mu}}{k^2}\{[\sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+k)^2)]S(p+k)\gamma_{\mu}(a-b\gamma_5)$$

$$+\gamma_{\mu}(a-b\gamma_5)S(p-k)[\sigma((p-k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]\}$$

$$+\frac{k^{\mu}}{k^2}\{\gamma_{\mu}(a-b\gamma_5)S(p+k)[\sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+[\sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p-k)^2)]S(p-k)\gamma_{\mu}(a-b\gamma_5)\}$$

$$-\frac{1}{k^2}\{[\sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+k)^2)]S(p+k)$$

$$\times[\sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+[\sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p-k)^2)]S(p-k)$$

$$\times[\sigma((p-k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+2(a^2+b^2)\sigma(p^2)-(a^2-b^2)[\sigma((p-k)^2)+\sigma((p+k)^2)]\}.$$

$$(5-23)$$

Using the on-shell condition in the chiral limit that

$$(\hat{p}+\sigma(p^2))u(p) = \bar{u}(p)(\hat{p}+\sigma(p^2)) = 0 \qquad (5-24)$$

we find upon substitution of (5-23) into (5-22) that for the Z sector's contribution to the induced fermion Yukawa (current) mass ($m^{ind} = m_Z^{ind} + m_W^{ind}$) is given by

$$-\frac{2b(m_Z^{ind})}{M_Z}$$

$$=4bM_Z\int\frac{d^4k}{i(2\pi)^4}\bar{u}(p)\{\gamma^{\mu}(a-b\gamma_5)\frac{-\hat{p}-\hat{k}+\sigma((p+k)^2)}{(p+k)^2-\sigma^2((p+k)^2)}\gamma_{\mu}(a-b\gamma_5)$$

$$+\frac{1}{k^2}[\sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma((p+k)^2)]\frac{-\hat{p}-\hat{k}+\sigma((p+k)^2)}{(p+k)^2-\sigma^2((p+k)^2)}$$

$$\times[\sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\sigma(p^2)]$$

$$+\frac{1}{k^2}[(a^2+b^2)\sigma(p^2)-(a^2-b^2)\sigma((p+k)^2)]\}u(p)\frac{1}{(k^2-M_Z^2)^2}. \qquad (5-25)$$

To evaluate (5-25), we first assume that the external self-energy $\sigma(k'^2)$ falls sufficiently quickly with k'^2 that integrals of the form $\int d^4k'\sigma(k'^2)F(k')$ can be neglected relative to $\sigma(p^2)\int d^4k'F(k')$ when $F(k')$ is negligible at small k'. We utilize the expansion

$$(k'^2-\sigma(k'^2))^{-1}=\frac{1}{k'^2}+\frac{1}{k'^2}\sigma(k'^2)\frac{1}{k'^2}+\quad\ldots$$

and then note that the nonleading terms in the expansion, upon integration over k', are suppressed relative to the leading term by factors of σ^2/M_Z^2. Consequently, we find that the leading contributions to m_Z^{ind} are given by

$$-\frac{2bm_Z^{ind}}{M_Z}$$

$$= 4bM_Z\bar{u}(p)\{(a^2+b^2)\sigma(p^2)\int\frac{d^4k'}{i(2\pi)^4}\frac{1}{k'^2(k'^2-M_Z^2)^2}$$

$$+\sigma^2(p^2)\int\frac{d^4k'}{i(2\pi)^4}\frac{(a-b\gamma_5)\not{k}'(a+b\gamma_5)}{k'^2((k'-p)^2-M_Z^2)^2(k'-p)^2}$$

$$+\int\frac{d^4k'}{i(2\pi)^4}\frac{(a+b\gamma_5)\gamma^\mu(-\not{k}')\gamma_\mu(a-b\gamma_5)}{k'^2((k'-p)^2-M_Z^2)^2}\}u(p)+O(\sigma^3/M_Z^3). \qquad (5-26)$$

The contribution of the first and third integral in (5-26) cancels upon utilization of the on shell condition (5-24) to replace $\not{p}u(p)$ (from evaluation of the third integral) with $-\sigma(p^2)u(p)$. The second integral yields a contribution that is of order σ^3/M_Z^3 in magnitude. Thus we find that the mass generated in the chiral limit by the induced Yukawa interaction of fig. 22's Z exchange graph is (at most) of order

$$m_Z^{ind} \approx |\sigma|^3/M_Z^2 < 10^{-2}\text{Mev} \qquad (5-27)$$

where $|\sigma|$ is assumed comparable to a dynamical mass (2-22) of order 300 Mev arising from the chiral noninvariance of the QCD vacuum.

The mass m_W^{ind} induced via fig. 22's W exchange can be shown to be characterized by the small upper bound of (5-27). This result, in and of itself, is disappointing; it might have been interesting had there been a causal connection demonstrated between the relatively small (~5 Mev) up and down quark current masses and the $m_{nucl}/3$ dynamical mass scale characterizing quark masses in static hadron processes. However, as the origin of the nonperturbative self-energy σ can be a theory with a much larger scale than QCD (technicolor for example), the method of this section may nevertheless be the correct mechanism for generating the current mass for up and down quarks.

5-5) Summary:

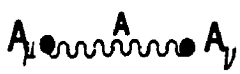
Gauge invariant electroweak calculations in the presence of nonperturbative QCD vacuum are possible only if its Green's functions are modified according to $SU(2)XU(1)$ Ward identities. These corrections of the electroweak Green's functions can be avoided if one uses Landau gauge. On the other hand, the physical ramifications of these modifications could be significant such as the problem of induced Yukawa coupling that was investigated in this chapter. Another problem under investigation is the calculation of the hadronic contribution to the muon magnetic moment using the quark model. It is known that current and constituent quark masses result respectively in too big and too small hadronic contributions. To improve this we propose to insert a momentum dependent self-energy and consequently correct the photon vertices accordingly. This problem is currently under investigation.

APPENDIX 1

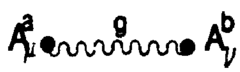
NOTATIONS AND FEYNMAN RULES

We follow the notations of the text book by Bjorken and Drell. Thus the metric is $g^{\mu\nu} = (1,-1,-1,-1)$ and γ_5 is defined by $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Thus Following is the list of the Feynman rules used in the text[37]. These rules are based on the lagrangians (1-25) and (1-27a).

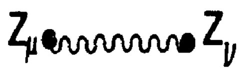
$$\frac{1}{m - \not{k}}$$



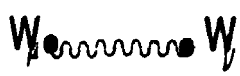
$$\left[\frac{g_{\mu\nu}}{k^2+i\varepsilon} - (1-\xi_A)\frac{k_\mu k_\nu}{(k^2+i\varepsilon)^2}\right]$$



$$\delta^{ab}\left[\frac{g_{\mu\nu}}{k^2+i\varepsilon} - (1-\xi_g)\frac{k_\mu k_\nu}{(k^2+i\varepsilon)^2}\right]$$



$$\frac{1}{k^2+i\varepsilon-M_Z^2}\left[g_{\mu\nu} - (1-\xi_Z)\frac{k_\mu k_\nu}{k^2+i\varepsilon-\xi_Z M_Z^2}\right]$$

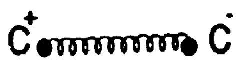


$$\frac{1}{k^2+i\varepsilon-M_W^2}\left[g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2+i\varepsilon-\xi_W M_W^2}\right]$$

$$\frac{1}{\xi_W M_W^2 - k^2 - i\varepsilon}$$

$$\frac{1}{\xi_Z M_Z^2 - k^2 - i\varepsilon}$$

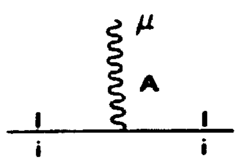
$$\frac{1}{m_\phi^2 - k^2 - i\varepsilon}$$



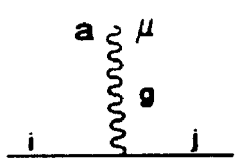
$$-\frac{1}{\xi_W M_W^2 - k^2 - i\varepsilon}$$

$$\frac{1}{\xi_Z M_Z^2 - k^2 - i\varepsilon}$$

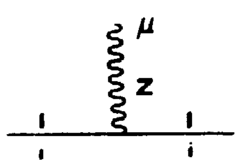
$$\frac{1}{-k^2 - i\varepsilon}$$



$$eQ_{l,i}\gamma^\mu$$



$$g_s \frac{\lambda^a_{ij}}{2}\gamma^\mu$$

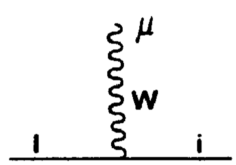


$$\gamma^\mu(a \mp b\gamma_5)$$

where a and b have the following values for up and down components of the doublet $\Psi = \begin{pmatrix} \Psi_l \\ \Psi_i \end{pmatrix}$.

$$a_{\begin{pmatrix} up \\ dn \end{pmatrix}} = \frac{eM_Z^2}{2M_W\sqrt{M_Z^2 - M_W^2}}\left((\pm)\frac{1}{2} + \begin{pmatrix} -4/3 \\ +2/3 \end{pmatrix}\frac{(M_Z^2 - M_W^2)}{M_Z^2} \right)$$

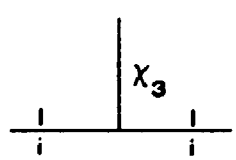
$$b_{\begin{pmatrix} up \\ dn \end{pmatrix}} = \frac{eM_Z^2}{4M_W\sqrt{M_Z^2 - M_W^2}}$$



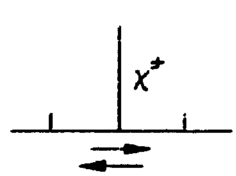
$$\frac{g}{2\sqrt{2}}U_{li}\gamma^{\mu}(1-\gamma_5)$$

where U_{li} is the element of mixing matrix. For two generations this matrix is written in terms of Cabbibo angle:

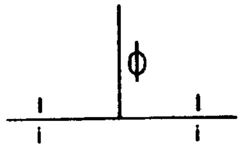
$$U = \begin{pmatrix} U_{ud} = \cos\theta_c U_{us} = \sin\theta_c \\ U_{cd} = -\sin\theta_c U_{cs} = \cos\theta_c \end{pmatrix}$$



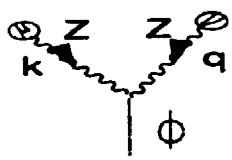
$$-i\frac{2bm_{l,i}}{M_Z}\gamma_5$$



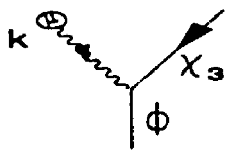
$$-i\frac{g}{2\sqrt{2}M_W}U_{li}\{(\pm m_l \mp m_i) + (m_l + m_i)\gamma_5\}$$



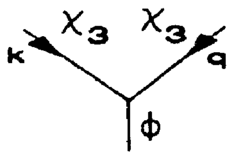
$$-\frac{2bm_{l,i}}{M_Z}$$



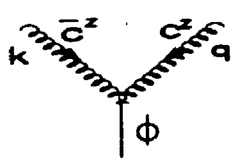
$$4bM_Zg^{\mu\nu}$$



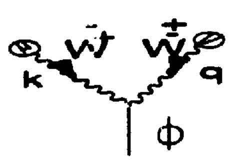
$$2ib(2q+k)^{\mu}$$



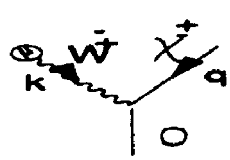
$$-\frac{2bm_{\Phi}}{M_Z}$$



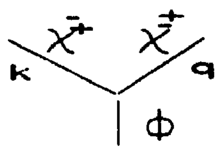
$$-\frac{2bM_Z}{\xi_Z}$$



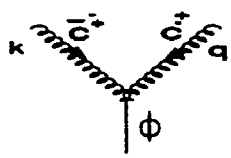
$$gM_Wg^{\mu\nu}$$



$$\frac{ig}{2}(2q+k)^{\mu}$$



$$-\frac{g m_\Phi^2}{2M_W}$$



$$-\frac{\xi_W g M_W}{2}$$

APPENDIX 2

BRST TRANSFORMATIONS

Following is the set of BRST transformations that leaves the electroweak lagrangian (1-25) invariant[37]:

$$\delta^{BRST}\Psi_l = i(a - b\gamma_5)C^Z\Psi_l + \frac{ig}{2\sqrt{2}}(1-\gamma_5)C^+\Psi_i + ieQ_lC^A\Psi_l$$

$$\delta^{BRST}\overline{\Psi}_l = -iC^Z\overline{\Psi}_l(a + b\gamma_5) - \frac{ig}{2\sqrt{2}}C^-\overline{\Psi}_i(1+\gamma_5) - ieQ_lC^A\overline{\Psi}_l$$

$$\delta^{BRST}\Psi_i = i(a + b\gamma_5)C^Z\Psi_i + \frac{ig}{2\sqrt{2}}(1-\gamma_5)C^-\Psi_l + ieQ_iC^A\Psi_i$$

$$\delta^{BRST}\overline{\Psi}_i = -iC^Z\overline{\Psi}_i(a + b\gamma_5) - \frac{ig}{2\sqrt{2}}C^+\overline{\Psi}_l(1+\gamma_5) - ieQ_iC^A\overline{\Psi}_i$$

$$\delta^{BRST}W^{\pm}_{\mu} = \partial_\mu C^{\pm} \pm \frac{ig}{\sqrt{g^2+g'^2}}[W^{\pm}_{\mu}(gC^Z + g'C^A) - (gZ_\mu + g'A_\mu)C^{\pm}]$$

$$\delta^{BRST}Z_\mu = -\frac{ig^2}{\sqrt{g^2+g'^2}}(W^+_\mu C^- - W^-_\mu C^+) + \partial_\mu C^Z$$

$$\delta^{BRST}A_\mu = -\frac{igg'}{\sqrt{g^2+g'^2}}(W^+_\mu C^- - W^-_\mu C^+) + \partial_\mu C^A$$

$$\delta^{BRST}\Phi = -\frac{g}{2}(\chi^+C^- + \chi^-C^+) - \frac{\sqrt{g^2+g'^2}}{2}\chi_3C^Z$$

$$\delta^{BRST}\chi^{\pm} = \frac{g}{2}[(v+\phi)C^{\pm} \mp \chi_3 C^{\pm}] \pm \frac{i}{2\sqrt{g^2+g'^2}}\chi^{\pm}[(g^2-g'^2)C^Z + 2gg'C^A]$$

$$\delta^{BRST}\chi_3 = \frac{\sqrt{g^2+g'^2}}{2}(v+\Phi)C^Z - \frac{ig}{2}(\chi^+C^- - \chi^-C^+)$$

$$\delta^{BRST}\overline{C}^{\pm} = -\left(\frac{1}{\xi_W}\partial^{\mu}W_{\mu}^{\pm} + M_W\chi^{\pm}\right)$$

$$\delta^{BRST}\overline{C}^{Z} = -\left(\frac{1}{\xi_Z}\partial^{\mu}Z_{\mu} + M_Z\chi_3\right)$$

$$\delta^{BRST}\overline{C}^{A} = -\frac{1}{\xi_A}\partial^{\mu}A_{\mu}$$

In order to balance the Grassmann type variables, it is understood that the ghost fields in the above equations are multiplied by a Grassmann constant.

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